

APPLICATION OF RECONSTRUCTION TOMOGRAPHY IN TWO-PHASE FLOW STUDIES

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for the Degree of
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**By
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ABSTRACT

The technique of reconstruction tomography used in radiology is applied to measure density in bubbly air-water flows. The 'Convolution' algorithm used, converts the fan beam data to a parallel beam data and then a suitable filter is used. (in our case the generalise filter). Then convolution is carried out, followed by superimposition. Printing of density values is done in the format of density maps. The Generalised filter used proved superior to the Shepp and Logan filter which was used as a reference.

The error band in the measured values of density is -0.03 and its underestimation is fairly constant and predictable within the range of ρ 0.6 to 1.0 .

The New Algorithm is an alternate method of reconstructing densities but the C.P.U time is more. The error was practically zero, for a certain class of density distribution called radially symmetric distribution.

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CHAPTER 1

INTRODUCTION

One of the major problems confronting a Nuclear reactor safety engineer, is the measurement of void fraction or density during Loss-Of-Coolant-Accident (LOCA). The development of an accurate and reliable two-phase flow measurement technique for various flow systems and components in light water reactors is of great importance and significance. The present study looks at such a new vista, which will alleviate the problems in the chemical industry, food processing, waste treatment and fluidized flow systems, including fluidized bed combustors. The method used is that of computerised axial tomography.

The most familiar images are those formed directly by optical instruments using visible light reflected or transmitted by an object. However, in many applications in which an image is required, we can make only indirect measurements by probing the object with invisible radiation or by interpreting it. Often the measured data is not in a

form suitable for immediate interpretation, but is related to the image in a known way. The general aim of all image reconstruction procedures or algorithms is to process the data to form an image and so to facilitate the interpretation of measurements.

To determine the density of the object under test various strip integrals corresponding to a particular angle of view were taken and the set of strip integrals are called projection of the object. Given a number of such projections at different angles of view, the estimation of the corresponding distribution within the object is the basic problem of image reconstruction from projections. Computerized Axial Tomography (CAT) is the most significant application to date of image reconstruction from projections.

According to Webster, 'TOMOS' means cutting and graphy is study and together mean a special X-ray/Gamma ray technique by which detailed images of a structure lying in a specific layer of tissue may be obtained, with images of structures in other layers eliminated or blurred.

A basic and crude form of CT (computerized tomography) was practised in Japan as early as 1947 and called as Rotatography by taking several X-ray images at different angles ($0-360^\circ$) around the patients body.

The practice now days is to use a detector to collect the data and process using a computer, which has the capability to detect minute differences in X-ray/Gamma ray attenuation coefficients, instead of directly using X-ray pictures (Radiograms) as was done with Rotatography.

The present study involves "Convolution" algorithm. This algorithm is considered as highly efficient by numerous investigators in computerised axial tomography. The reconstruction method in use in all CTs is the Convolution method.

The above method has been aptly been demonstrated by Kulacki et al (1). Reconstruction tomography was adapted to the needs of two-phase flow density measurements. Design of a prototype scanning densitometer has also been suggested by Schlosser et al (2).

There are several parameters in the Convolution algorithms and becomes implementation dependent. The present study considered all the free parameters and their effect on the problem in question. The parameters considered are viz:-

1. Number of scans - N -
2. Number of beams in a scan - M - and
3. Various filter functions

The reconstruction algorithm used is that of Kwok et al (3) with the Shepp and Logan filter. The present study investigates other possible filters like Ramachandran filter, Parabolic filter and Generalised filter.

The present study has been undertaken with the main objective to improve the reconstruction algorithm used in the work of Kulacki et al (1). Especially the filter function (Shepp and Logan) used has been carefully investigated.

A FORTRAN program for the algorithm has been written and implemented with flexibility to change number of Rays per view and number of projections in scan as per the computer time and accuracy needed.

5

Specific objectives of the study are:-

1. To find the optimum values for number of Rays per projection (LITM).
2. To find the optimum values for number of projections in scan (N) and
3. To find the best filter.

LITM was checked for two values 20 and 40 .

N was checked for three values 10, 20 and 40.

Parabolic filter, Ramachandran filter and four cases of Generalized filter were investigated, graphs plotted. In this test for best possible filter Calibration curves were drawn for a few known densities of material like Air, Water, Walnut and Pine.

The data used in all the computations have been taken from Kulacki et, al (1).

A 'New Algorithm' was investigated to get exact reconstruction. This was developed by Prof.R.K.S.Rathore. The data used were test values. Experimental Data is to be adopted for this method.

CHAPTER-2

LITERATURE REVIEW

2.1 OVER VIEW

One of the complex situations to analyse in Nuclear reactor safety is the two-phase flow condition and void fractions formed in them. There are various methods used for this purpose and Kulacki et al (1) have amply and sufficiently proved in their study the use of scanning gamma-ray densitometer. There are no disturbance caused which are inevitable if probes are inserted in the flow for point density measurements. Also a scanning gamma-ray densitometer offers the potential for obtaining both point and cross-section averaged values of density without disturbing the flow field (because of invasive technique). Most of the present study is based on the technique involved and developed by (1).

The other prominent in the category of invasive type of measurement of void fraction are:-

Nuclear Magnetic Resonance (NMR) and

Ultrasonics;

Computerised axial tomography (CAT) the method adapted to two-phase flow in Nuclear reactors by Schlosser et al (2) with the aid of Kwok et al (3) algorithm is explained in the following pages.

Ramachandran et al (5) has explained in their study a simplification of reconstruction by using convolution instead of Fourier transform, which greatly reduces the computer time and space needed alongst with a better accuracy.

2.2 RAMACHANDRAN FILTER

This new technique is used for 3D reconstruction from their transmitted radiographs. Convolution is used in the real space of the object, without using Fourier Transforms (F.T's). The object is rotated about an axis at right angles to the direction of a parallel beam of radiation, and sections of it normal to the axis are reconstructed from Data obtained by scanning the corresponding linear strips in the shadowgraphs at different angular settings.

Since convolution method involves only one summation over one variable at a time, while F.T requires two, convolution is much faster. It is also shown by tests that convolution method is more accurate.

The basic step in the solution of reconstruction problem consists of replacing the three dimensional density distribution by a set of two-dimensional density functions in a series of sections perpendicular to Z-axis.

$$F(R; \theta) = \int_{-\infty}^{\infty} g(l; \theta) \exp(2\pi i R l) dl \dots\dots\dots(1)$$

$$f(r; \phi) = \int_0^{2\pi} \int_0^{\infty} F(R; \theta) \exp(-2\pi i R r \cos(\phi - \theta)) R dR d\theta \dots\dots(2)$$

Suppose we define:

$$g^1(l, \theta) = \int_{-\infty}^{\infty} |R| F(R; \theta) \exp(-2\pi i R l) dR \dots\dots\dots(3)$$

Equation (2) for $f(r, \phi)$ becomes

$$f(r, \phi) = \int_0^{\pi} g(r \cos(\phi - \theta), \theta) d\theta \dots\dots\dots(4)$$

Fourier inverting (1)

$$g(l; \theta) = \int_{-\infty}^{\infty} F(R; \theta) \exp(-2\pi i R l) dR \dots\dots\dots(5)$$

$$F.T \text{ of } g(l; \theta) = (F.T. \text{ of } g(l; \theta)) \times (F.T. \text{ of } q(l))$$

R is the F.T. of the function $q(l)$.

$$|R| = \int_{-\infty}^{\infty} q(l) \exp(2\pi i R l) dl \dots\dots\dots(6)$$

Using convolution theorem for the inverse of the product of F.T's

$$g'(l; \theta) = \int_{-\infty}^{\infty} g(l_1; \theta) q(l-l_1) dl_1 \quad \dots\dots(7)$$

$$q(l) = - \int_{-\infty}^{\infty} |R| \exp(-2\pi i R l) dR \quad \dots\dots(8)$$

The integral diverges for the limits so replacing the limits by $-A/2$ and $+A/2$ where $A = 1/a$.

$$q_A(l) = \int_{-A/2}^{A/2} |R| \exp(-2\pi i R l) dR \quad \dots\dots(9)$$

$$q(na) = 1/4a^2 \quad \text{for } n=0$$

$$= -1/\pi^2 n^2 a^2 \quad \text{for } n \text{ odd} \quad \dots\dots(10)$$

$$= 0 \quad \text{for } n \text{ even}$$

$f(r, \phi)$ is the two dimensional density distribution.

$f(r, \phi, z)$ will be the data of three dimensional density distribution using all linear strips at Z in the different shadowgraphs taking at various angles θ .

$F(R; \theta)$ is the Fourier transform of $g(l; \theta)$

where $g(l; \theta; z)$ is the logarithm of the ratio of intensity at point $P(z, l)$ to the incident intensity. (which is a measure of the linear integral).

The biggest problem in this filter is that it is zero for all even values so the filter becomes highly problem specific.

2.3 SHEPP AND LOGAN FILTER

The study of shepp et al (6) .reveals that the Fourier reconstruction may be viewed simply in the spatial domain as the sum of each line integral times a weighting function of the distance from the line to the point of reconstruction.

Ramachandran's linear interpolation is merely the choice of a particular weighting function. A modified weighting function simultaneously achieves accuracy, simplicity, low computation time as well as low sensitivity to noise.

Using a simulated phantom, comparison studies were done on the Fourier algorithm and a search algorithm very similar but not identical to Housfields algorithm. The search algorithm requires 12 iterations to obtain a reconstruction of accuracy and resolution comparable to that of Fourier reconstruction and more sensitive to noise. To speed the search algorithm using fewer iterations results in error.

The steps similar to Ramachandran's filter derivation are done with the modification in the weighting function.

$$q(0) = 4/\pi a^2 \quad \text{for } K=0$$

$$q(Ka) = -4/\pi a^2 (4K^2 - 1) \quad \text{for } K = \pm 1, 2, \dots$$

This is the Shepp and Logan filter.

2.4 GENERALISED FILTER

In their study Kwok et al (4) have given a new filter called generalised filter. One general class of digital w - filter without interpolation is defined by

$$W(w) = \begin{cases} \frac{1}{a} |w| \exp \left\{ -\left| \frac{w}{w_c} \right|^p \right\} = \frac{1}{a} w \exp \left\{ -\left| \frac{w}{w_c} \right|^p \right\} & w \leq \pi/a \\ \frac{1}{a} \left| \frac{2\pi}{a} - w \right| \exp \left\{ -\left| \frac{2\pi}{a} - w \right|^p \right\} & \pi/a \leq w \leq \frac{2\pi}{a} \end{cases}$$

where $\zeta \left(\frac{w}{w_c} \right)$, $p \geq 0$; a is the sample spacing and $W(w)$ is the periodic with period $2\pi/a$.

$\zeta = \left| \frac{1}{w_c} \right|^p$ is called the damping factor. It is a function of the cut-off frequency w_c and the parameter p . the increased number of parameters over what was used in other filters makes it easier to fit a filter specification.

The portion $\exp \left(- \left| \frac{w}{w_c} \right|^p \right)$ serves as a switch.

At low frequencies, it is not triggered and $W(w) = |w|$.

For higher frequencies, it switches to zero as w approaches the cut-off frequency w_c . $W(w)$ tends to zero with a rate

depending on the parameters p and w_c . To minimise oscillations it is desirable to choose w_c and p so that a smooth cut off results. This is exactly what that is done. Four such values taken and tested to find which amongst them fit in and give the best of results for the problem in question.

When $\xi = 0$, the generalised filter is identical to the triangular wave filter used by Ramachandran. For certain values of ξ and p , the generalised filter approximates the filter of sheppard logan very (fairly) accurately. For $p > 1$ as in all the four cases considered numerical analysis is used to obtain the response.

$$W(na) = \frac{1}{\pi} \int_0^{\pi/a} w \exp - \xi w^p \cos naw \, dw \quad \dots\dots(11)$$

This is the response for $n = 0$

for other n 's. We change variables by putting $z = naw$.

So that (11) approximates to

$$W(na) = \frac{1}{\pi a^{1-p}} \int_0^{n\pi} z \exp - \xi \left(\frac{z}{an}\right)^p \cos z \, dz \text{ for } n=1,2,\dots \quad \dots\dots(12)$$

for $n=1$ we get,

$$W(a) = \frac{1}{\pi a^{1-p}} \int_0^{\pi} z \exp - \xi \left(\frac{z}{a}\right)^p \cos z \, dz \quad \dots\dots(13)$$

for $n > 1$, (12) is put in alternate form

when n is even

$$W(na) = \frac{1}{\pi a^2 n^2} \int_0^{2\pi} \cos z \left(\sum_{j=0}^{\frac{n}{2}-1} (z+2\pi j) \exp \frac{-(z+2\pi j)^p}{(2n)^p} \right) dz$$

when n is odd > 1

$$W(na) = \frac{1}{\pi a^2 n^2} \int_0^{2\pi} \cos z \left(\sum_{j=0}^{\frac{n-1}{2}} (z+2\pi j) \exp \frac{-(z+2\pi j)^p}{(2n)^p} \right) dz \\ + \frac{1}{\pi a^2 n^2} \int_0^{\pi} \cos z \left((z+2\pi \frac{n-1}{2}) \exp \frac{-(z+2\pi \frac{n-1}{2})^p}{a n^2} \right) dz$$

$$W(na) = W(-na) \text{ for } n = 1, 2, \dots$$

This filter in turn was used with various values for its parameters p and $Zeta$, which were recommended by Kwok et al.(4).

$P = 2.03399$ and $Zeta = 2.243E-06$ is the approximation of a generalised filter to Shepp and Logan filter.

$P = 1.2$ and $Zeta = 5E-04$ is the approximation between Shepp and Logan and parabolic filter.

$P = 2.0$ and $Zeta = 3E-05$ best minimized the internal undershoot while preserving the shape of the object.

$P = 2.6$ and $Zeta = 0.96E-05$ was also used.

2.5 PARABOLIC FILTER

The parabolic filter is an approximation of certain parameters of a generalised filter and particular value of p and Zeta ($\frac{1}{\zeta}$).

The filter has response of

$$W(0) = \frac{\pi}{3a^2} \quad \text{for } n=0$$

$$W(na) = \frac{-1}{\pi(na)^2} \quad \text{for } n = \pm 1, \pm 2 \dots\dots\dots$$

$$W(w) = \frac{1}{a} \left\{ |w| - \frac{a}{2\pi} w^2 \right\} \quad 0 \leq w \leq \frac{2}{a}$$

is the frequency response of a parabolic filter.

2.6 Filter used :

Three dimensional (3D) reconstruction for diverging X-ray/Gamma ray beams as studied by Kwok et al (7) has shown that fast digital convolution technique is extended to diverging X-ray/Gamma ray beams by suitable changes to parallel X-ray/Gamma ray beams conditions. This along with the generalised $|w|$ filter yields good accuracy and excellent results. It also has a flexibility to cope with noise and substantial reduction in X-ray dosage.

The background for this work is based on Bracewell and Riddle who first suggested Fourier Transform (F.T) might be used to reconstruct an object and that X-ray scanning replaced 3D object by a series of 2D cross-section of the object. Then Ramachandran and Laxminarayan's work and their significance already discussed have improved the situation further by introducing convolution.

The key equations are

$$f(x,y) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |w| dw p(w,\theta) \exp(iw(x\cos\theta + y\sin\theta)) \quad \dots(14)$$

w is spatial frequency and $p(w,\theta)$ is F.T. of

projection data for parallel beams.

$$b(t,\theta) = \frac{1}{2\pi} \int_{-\infty}^{\infty} |w| p(w,\theta) \exp(iwt) dw \quad \dots(15)$$

$t = x\cos\theta + y\sin\theta$, $b(t,\theta)$ is called the back projection

formula. By the convolution property of the F.T

$$b(t,\theta) = \int_{-\infty}^{\infty} p(\xi,\theta) w(t-\xi) d\xi \quad \dots(16)$$

where $p(t,\theta)$ and $w(t)$ are respectively inverse F.T of $p(w,\theta)$ and $|w|$.

The inverse F.T of $|w|$ does not converge so (16) is valid only in a limiting sense. Since most of the objects have spatial frequency of interest only below some cut-off

frequency w_c , we can replace w with a function $W(w)$ which approximate $|w|$ for $w < w_c$ and zero for $w > w_c$. The class of functions $W(w)$ with these properties are called $|w|$ functions. This is exactly the generalised filter function.

CHAPTER-3

PROGRAM DESCRIPTION

3.1 Overview:

The application of the concept of computerized tomography scanning in two-phase flow problems results in absolute point density values in the pipe cross section averaged over time. The program involves four basic steps which include data input, adjustment of data, reduction and finally the reconstruction of the density map.

The data were taken from Kulacki et al (1) and was in the form of a fan beam geometry. Data was taken at every 2.5 degrees for a total of 50 degrees. The normalised counts were denoted by $h(\lambda, \beta)$, refer Fig. 3.1

where,

$$\beta = i\alpha \quad \text{for } 0 \leq i \leq N$$

$$\lambda = Sa \quad \text{for } -\frac{M}{2} \leq S \leq \frac{M}{2},$$

M is the number of diverging beams in the fan

N is the number of scans

$$a = 2\lambda_{\max} / M,$$

$$\text{and } \alpha = \pi / N$$

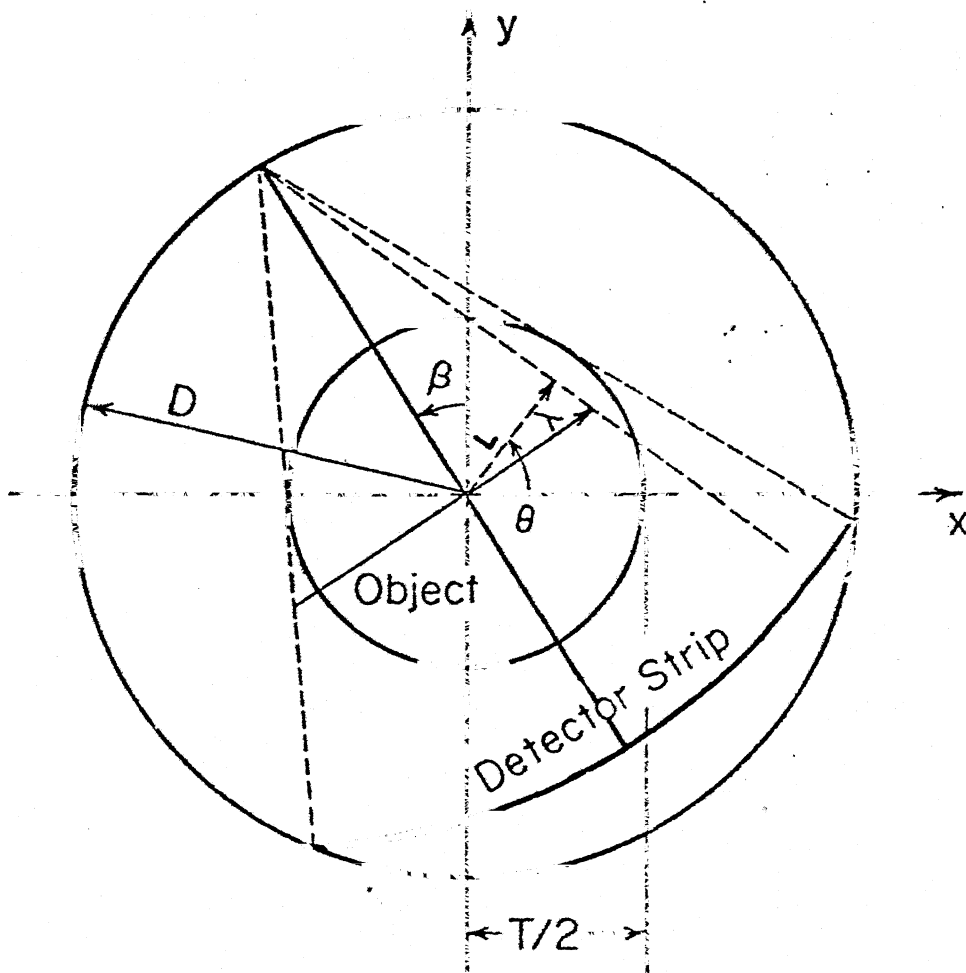


Figure 3.1 Conversion of Fan-Beam Data to Parallel Geometry.

Since the tomography concept is applicable only to the parallel beam case, the fan beam data has to be transformed. The transformed input, $p(L, \theta)$ was determined

$$\text{by } \lambda = \frac{1}{\sqrt{1-(L/D)^2}} \quad \beta = \theta - \sin^{-1}(L/D), \quad 3.1$$

$$\text{and } p(l, \theta) = h(\lambda, \beta) = h\left(\frac{1}{\sqrt{1-(L/D)^2}}, \theta - \sin^{-1}\left(\frac{L}{D}\right)\right) \quad 3.2$$

equation (3.2) involves the lengths L, D and λ , and angles θ and β , which are defined in Fig. (3.1).

The third step involves the use of a filter function, W and the computation of the convolution C (1a).

$$C(La) \triangleq \sum_{K=-\frac{M}{2}}^{\frac{M}{2}} p(t_k, \theta) W(t_L - t_k)$$

where,

θ_i is the j^{th} angular position

$$t_i = ia \quad \text{for } i = j, k, L,$$

j, k, L are all integers,

$$t_j = X \cos \theta_j + Y \sin \theta_j$$

Once $C(t_L, \theta_i)$ is known, the back projection, b and can be determined.

$$b(t, \theta_i) = a \frac{t_{L+1} - t}{t_{L+1} - t_L} C(t_L, \theta_i) + \frac{t - t_L}{t_{L+1} - t_L} C(t_{L+1}, \theta_i)$$

and finally the reconstructed density function $f(x, y)$ is

$$f(x, y) = \frac{1}{2N} \sum_{i=0}^{N-1} b(t, \theta_i), \quad 0 \leq i \leq N-1$$

The print out of the program is enclosed in

Appendix A

3.2 Steps involved :

The essential steps involved were as follows :

The input data was given in the proper format along with source to center distance (D), diameter of the object (T) Number of scans and number of projections in scan (M), etc. Then this was supplemented with LITM and N and a conversion to parallel beam done, along with interpolation to increase the number of data points.

Then the next step was in choosing the filter function. Schlosser et al (2) in their study have amply proved that Shepp and Logan filter is simple and good enough to the data used. This is verified and Parabolic filter and four cases of generalised filter are also taken up.

The generalised filter was calculated using integration routines and this was done with a limited accuracy. There were four integrations to perform to calculate separately values for $N=0$, 1, odd and even.

This shows as to why generalised filter was not used by Kulacki et al (1). The calculation of generalised filter is really a cumbersome process involving four integrations. Four types of generalised filters were considered depending upon which other common types then approximated to. There were four values of p and ZETA chosen and they are as follows.

1. $P= 2.0$ and $Zeta= 1.07E-02$
2. $P= 2.0$ and $Zeta= 4.28E-02$
3. $P= 2.0$ and $Zeta= 4.75E-03$
4. $P= 2.0$ and $Zeta= 2.67E-03$

These values of filter were used and Convolution found. Then Superimposition and Printing done in the proper format and LITF directly gives density values.

3.4 Data used:

The Data was taken from the study conducted by Kulacki et al(1). The source of nuclear radiation was 13.86 mCi of Cs-137 and fixed. The Collimator fixed at the other end was rotated through an angle of 50° (25° on each side).

The Detector was connected to this and the readings taken through preamplifier, amplifier, baseline restorer, Single channel analyser and a timer.

The Data was collected for eight various materials. **Four** of them water, air, walnut and pine are used for calibration.purpose. The other four 90% density, 80% density, 70% density and 60% density were taken for six types of scans and calculated and drawn accordingly on the graph.

CHAPTER-4

RESULTS

4.1 THEORITICAL CONSIDERATIONS

Density measurements in two-phase flow via the techniques used in axial computerized Tomography involves the data output format, interpretation of output and calibration of the computer algorithm. In each area the problem of relating what the reconstruction calculation means both physically and mathematically must be addressed. The data obtained in the present study is interpreted and best utilized for the overall objective of the investigation.

INTERPRETATION OF DATA OUTPUT

A typical reconstruction output appears in Matrix form shown in Fig. 4.0. The elements of this output Matrix vary from 0 to 99. At the bottom of the picture are indicated the maximum and minimum values of 'LITF'. This variable, LITF, is related to point density values. In fact, each number corresponds to the density of unit cell around it. Linear interpolation is used to reduce

MINIMUM LITF= 0.39390406E+00 MAXIMUM LITF= 0.16877217E+01

These are the values for Scan Six 60% Density or 40% Volt ($\rho=0.6$) for $\rho_{2.0}$ and $\rho_{2.4}$ the new values(1)

UT 055

the actual values of LITF within a range of 0 to 99.

Since the quantify of interest is the horizontally averaged density, $\langle \rho \rangle$, the cross-sectional average of LITF is required. Thus $\langle \text{LITF} \rangle$ is the sum of the values of LITF over the cross-section divided by the number of unit cells present.

The algorithm output is a 40x40 Matrix while the, pipe wall effects prevents any reasonable values to be had above 31 units. Hence 31 x 31 Matrix is considered good and used for averaging and finding 'LITF'.

$$\text{LITF} = \frac{\sum_{i=1}^{961} \text{LITF}_i}{961} \quad \dots (4.1)$$

where 961 is (31x31) the total number of unit cells.

The reconstruction accuracy for Shepp and Logan filter is around $\pm 10\%$. The accuracy for generalized filter is around $\pm 4\%$.

CALIBRATION

The quantity $\langle \text{LITF} \rangle$ determined by equation (4.1) corresponds to the actual density values across the flow area. Scans for the cases of Water ($\rho=1.0$ g/cm³), Air (ρ -approximately zero g/cm³), Walnut ($\rho=0.73$ g/cm³) and Pine ($\rho=0.41$ g/cm³)

result in four different values of LITF for known densities. Since these known densities are accurate within $\pm 0.5\%$ a reasonably good plot of LITF verses p is achieved. Densities are read off directly from this calibration curve once LITF is known. The above calibration was used for six types of filters viz:-

1. Shepp and Logan filter
2. Parabolic filter and
3. Four cases of Generalised filter.

A comparison of calibration curve can be made and some interesting points noted.

Ramachandran filter was also used and found unsuitable for the present study. Similar observations were made by Kulacki et al (1) in their study of filters.

Figures (4.1) through (4.6) represent calibration for different filters used. Fig.(4.1) is for the Shepp and Logan filter which can be compared to that obtained by Schlosser et al (2). Fig. (4.2) represents calibration for parabolic filter and Fig.(4.3) to (4.6) for four different cases of Generalised filter (sec. 2.4).

Table 4.1 Calibration of LITF using Shepp & Logan Filter

Case	ρ	LITF
Water	1.0	1.075
Walnut	0.73	0.6986
Pine	0.41	0.351
Air	0.0	0.0966

Table 4.2 Calibration of LITF using Parabolic Filter

Case	ρ	LITF
Water	1.0	0.751
Walnut	0.73	0.480
Pine	0.4	0.222
Air	0.0	0.08

Table 4.3 Calibration of LITF using Generalised Filter

Case	ρ	Gen 1	Gen 2	Gen 3	Gen 4
Water	1.0	0.924	0.915	0.926	0.928
Walnut	0.73	0.736	0.728	0.682	0.74
Pine	0.41	0.489	0.483	0.491	0.492
Air	0.0	0.0	0.0	0.0	0.0

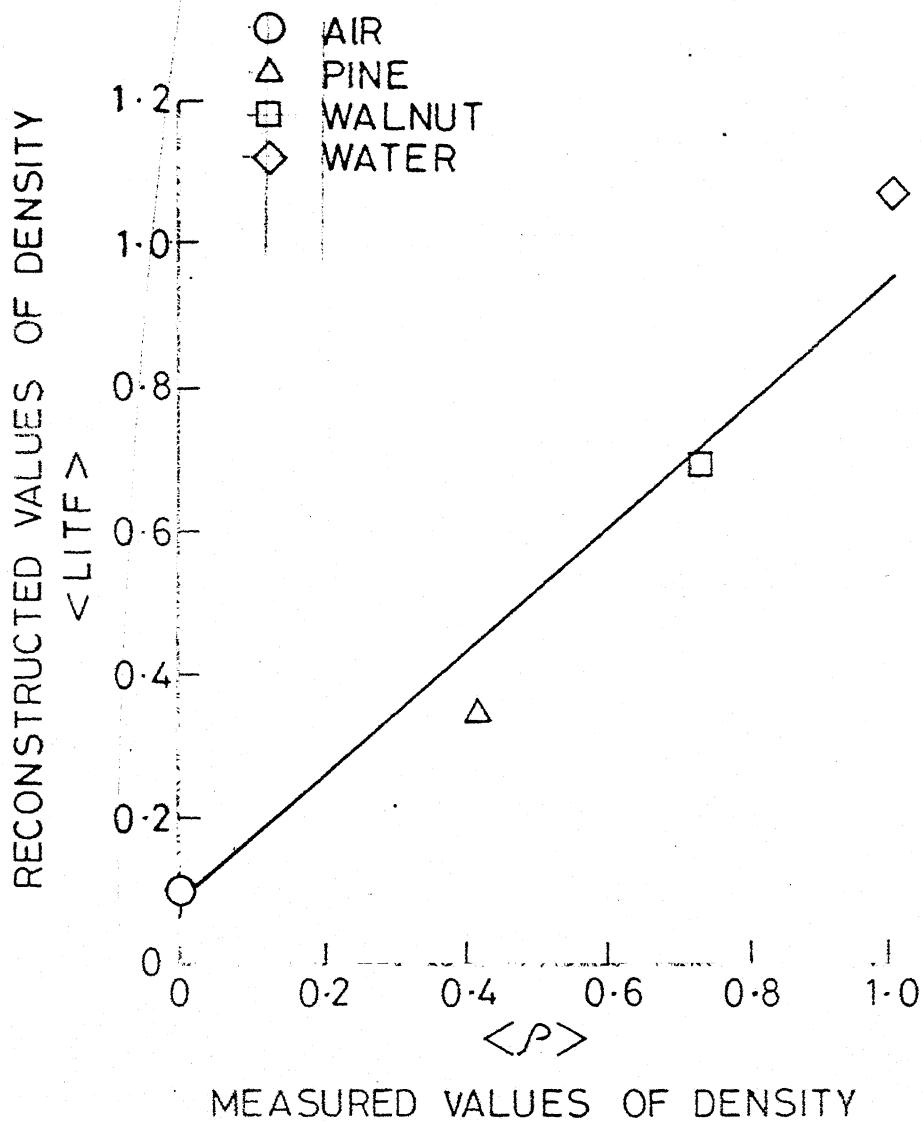


FIG. 4.1 CALIBRATION OF SHEPP AND LOGAN FILTER

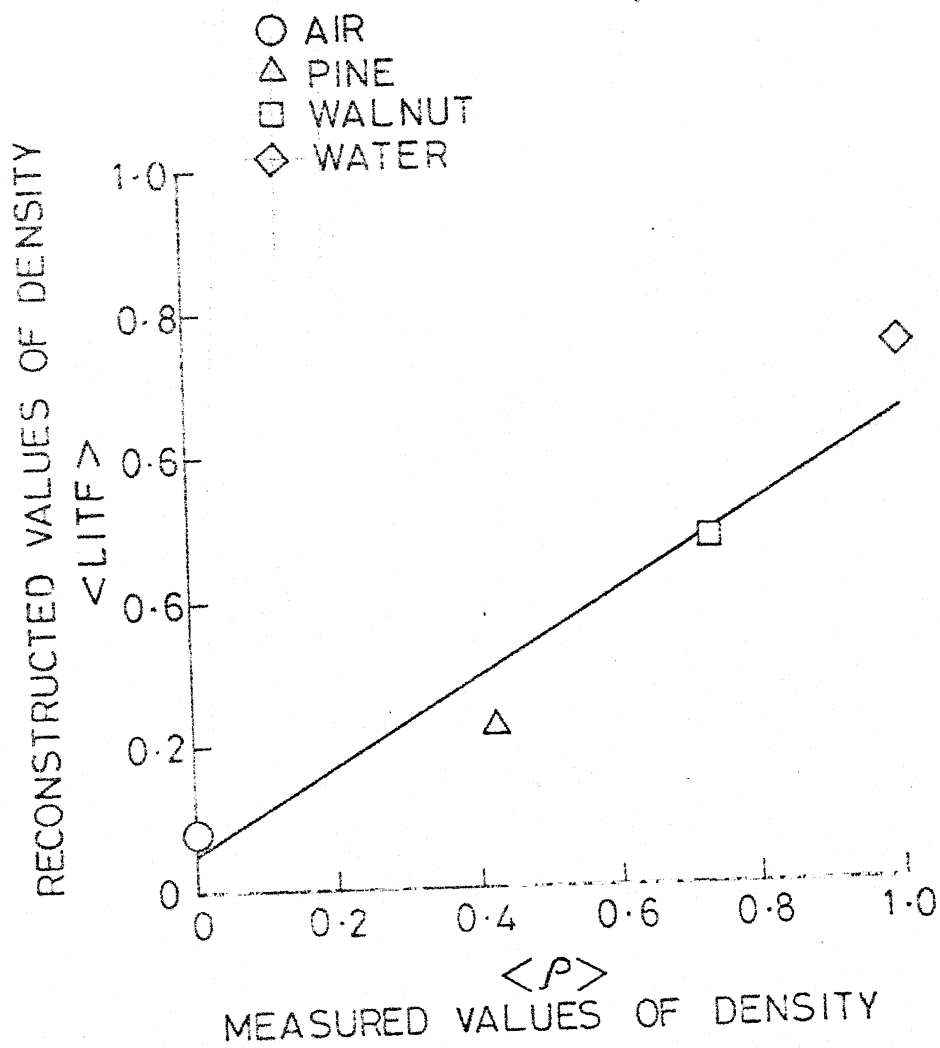


FIG. 4.2 CALIBRATION OF PARABOLIC FILTER

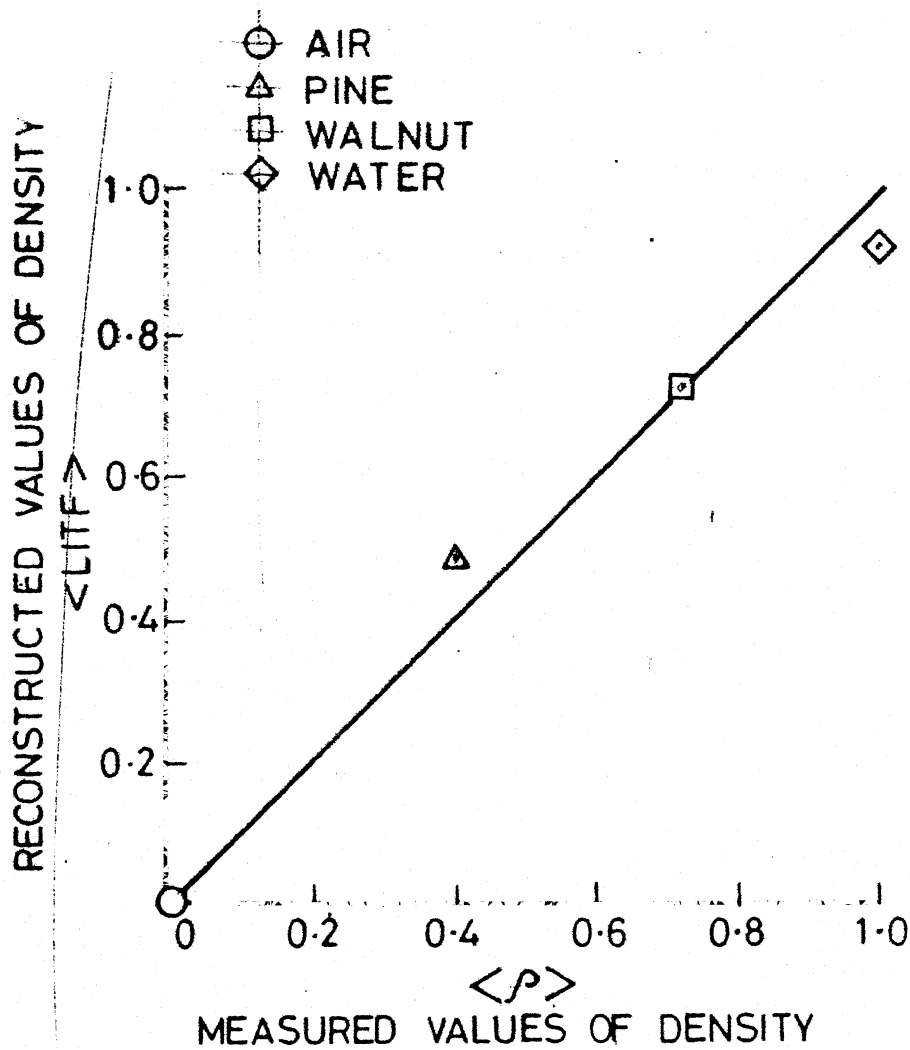


FIG. 4.3 CALIBRATION OF GENERALISED FILTER NO

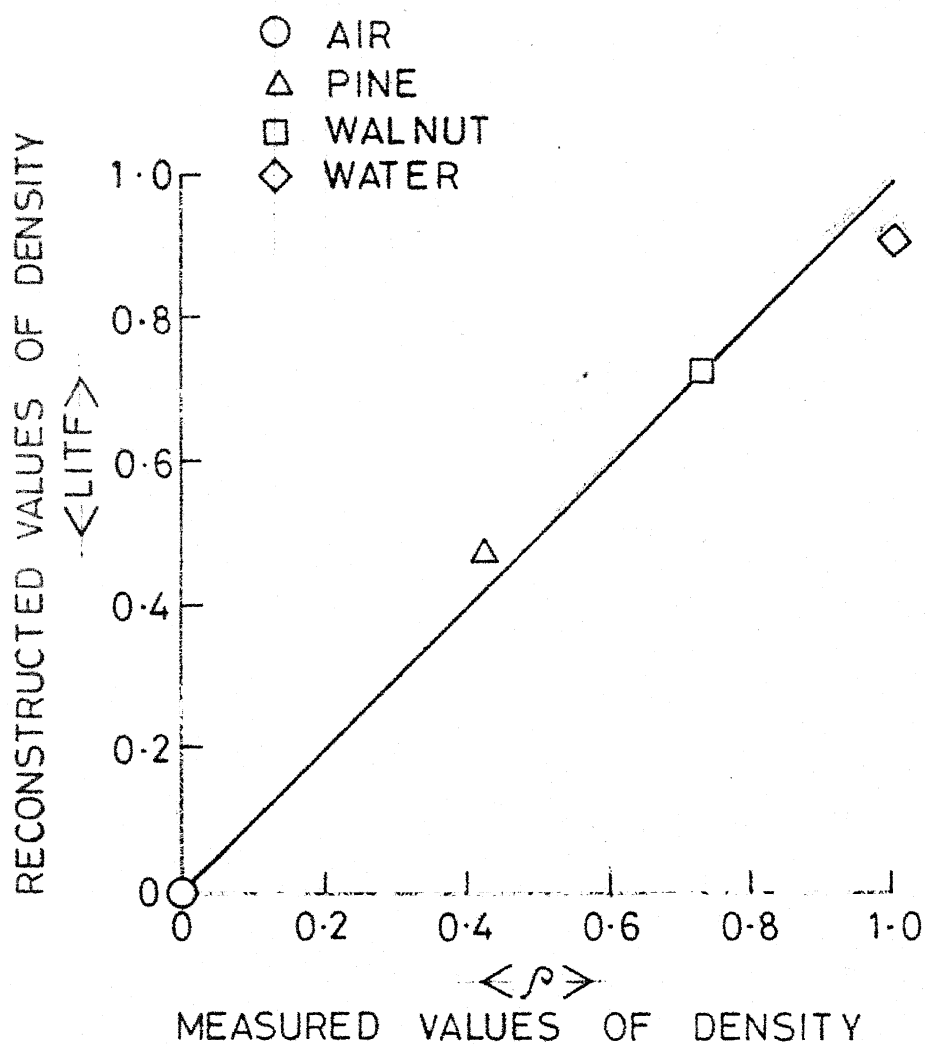


FIG. 4-4 CALIBRATION OF GENERALISED FILTER NO.

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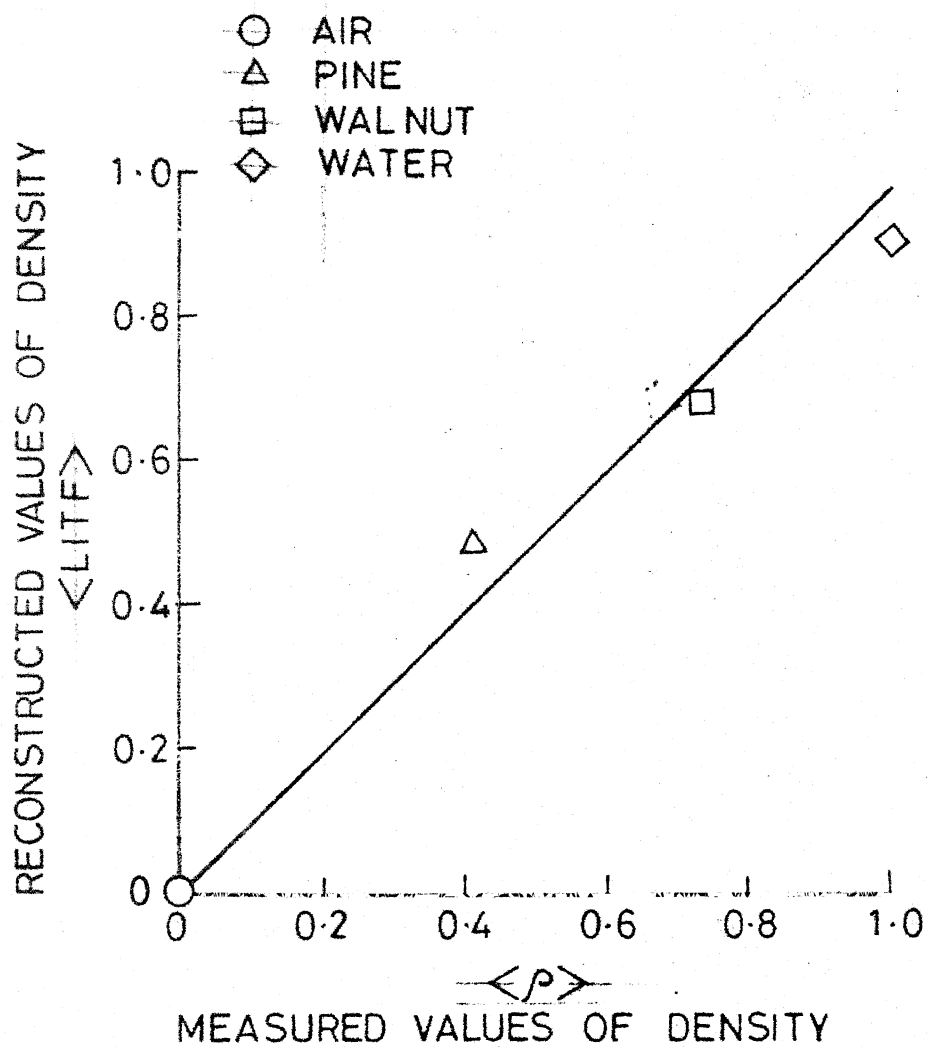


FIG. 4.5 CALIBRATION OF GENERALISED FILTER NO.3

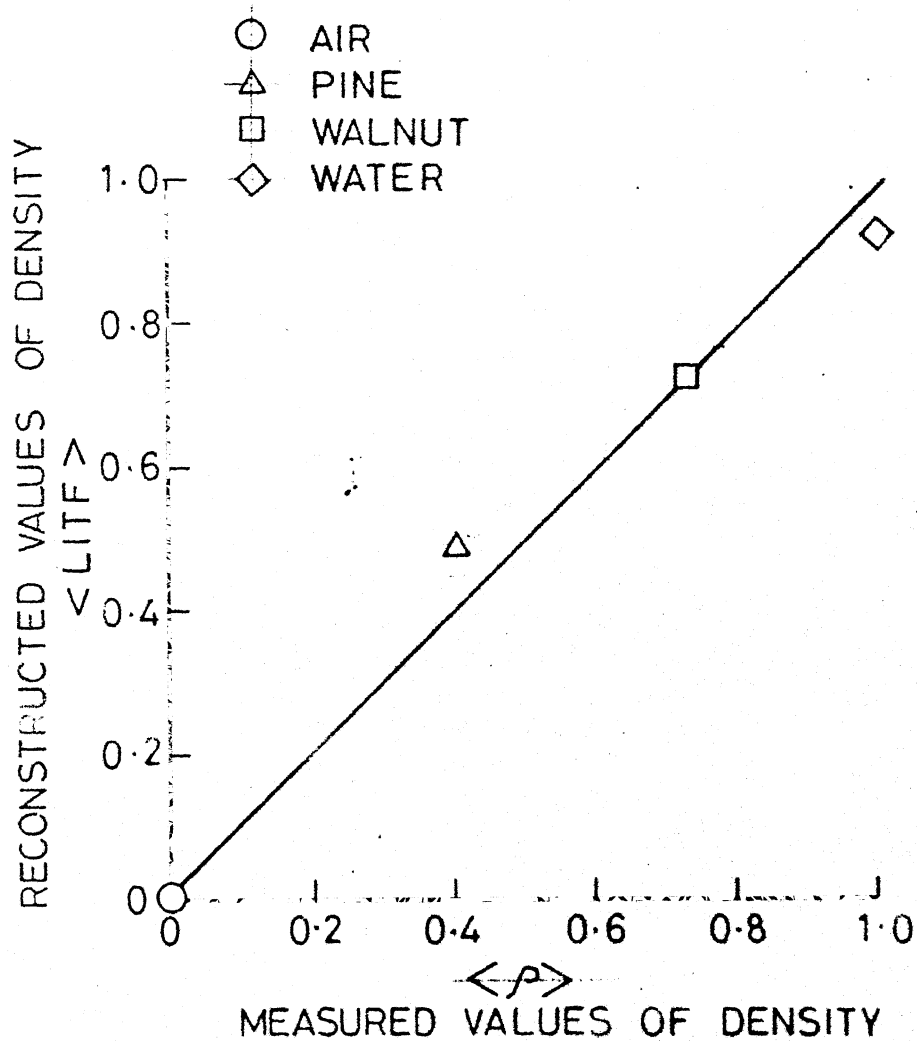


FIG.4-6 CALIBRATION OF GENERALISED FILTER NO.4

Table 4.4 Values of LITF for various scans for Shepp and Logan filter

Scan No. \ ρ	0.9	0.8	0.7	0.6	
1	1.15	0.97	0.93	0.71	
2	0.97	0.91	0.85	0.74	
3	1.02	1.05	0.83	0.90	Rejected
4	1.01	0.96	0.88	0.74	
6	1.23	1.03	0.86	0.73	

Table 4.5 Values of LITF for various scans for Parabolic Filter

Scan No. \ ρ	0.9	0.8	0.7	0.6	
1	0.84	0.7	0.69	0.52	
2	0.69	0.66	0.62	0.55	
3	0.73	0.77	0.61	0.68	Rejected
4	0.72	0.7	0.66	0.56	
6	0.91	0.76	0.63	0.54	

Table 4.6 Values of LITF for various scans for Generalised
Filter Number 1

Scan No. \ ρ	0.9	0.8	0.7	0.6
1	0.82	0.76	0.684	0.573
2	0.81	0.744	0.67	0.57
3	0.813	0.74	0.643	0.564
4	0.824	0.75	0.66	0.543
6	0.84	0.77	0.694	0.584

Table 4.7 Values of LITF for various scans for Generalised
Filter number 2

Scan No. \ ρ	0.9	0.8	0.7	0.6
1	0.81	0.75	0.68	0.57
2	0.804	0.74	0.66	0.56
3	0.81	0.74	0.64	0.56
4	0.82	0.74	0.65	0.54
6	0.83	0.76	0.69	0.58

Table 4.8 Values of LITF for various scans for Generalised
Filter number 3

ρ Scan No.	0.9	0.8	0.7	0.6
1	0.823	0.76	0.69	0.58
2	0.82	0.76	0.672	0.57
3	0.82	0.75	0.65	0.57
4	0.825	0.755	0.663	0.55
6.	0.84	0.775	0.7	0.59

Table 4.9 Values of LITF for various scans for Generalised
Filter number 4

ρ Scan No.	0.9	0.8	0.7	0.6
1	0.83	0.765	0.69	0.58
2	0.82	0.753	0.676	0.574
3	0.823	0.75	0.652	0.572
4	0.833	0.76	0.667	0.546
6	0.843	0.779	0.70	0.59

Table 4.10 Errors obtained for different scans (Gen 4)

Scan No.	$\langle \beta \rangle = 0.9$	$\langle \beta \rangle = 0.8$	$\langle \beta \rangle = 0.7$	$\langle \beta \rangle = 0.6$
1	0.07	0.03	0.01	0.02
2	0.08	0.05	0.02	0.02
3	0.08	0.05	0.05	0.03
4	0.07	0.03	0.03	0.04
6	0.06	0.02	0.0	0.01

Table 4.11 Relative errors Vs ρ (Gen 4)

$\langle \rho \rangle$	$\Delta \langle \rho \rangle$	$\Delta \langle \rho \rangle / \langle \rho \rangle$
0.9	0.072	0.08
0.8	0.036	0.045
0.7	0.023	0.033
0.6	0.02	0.030

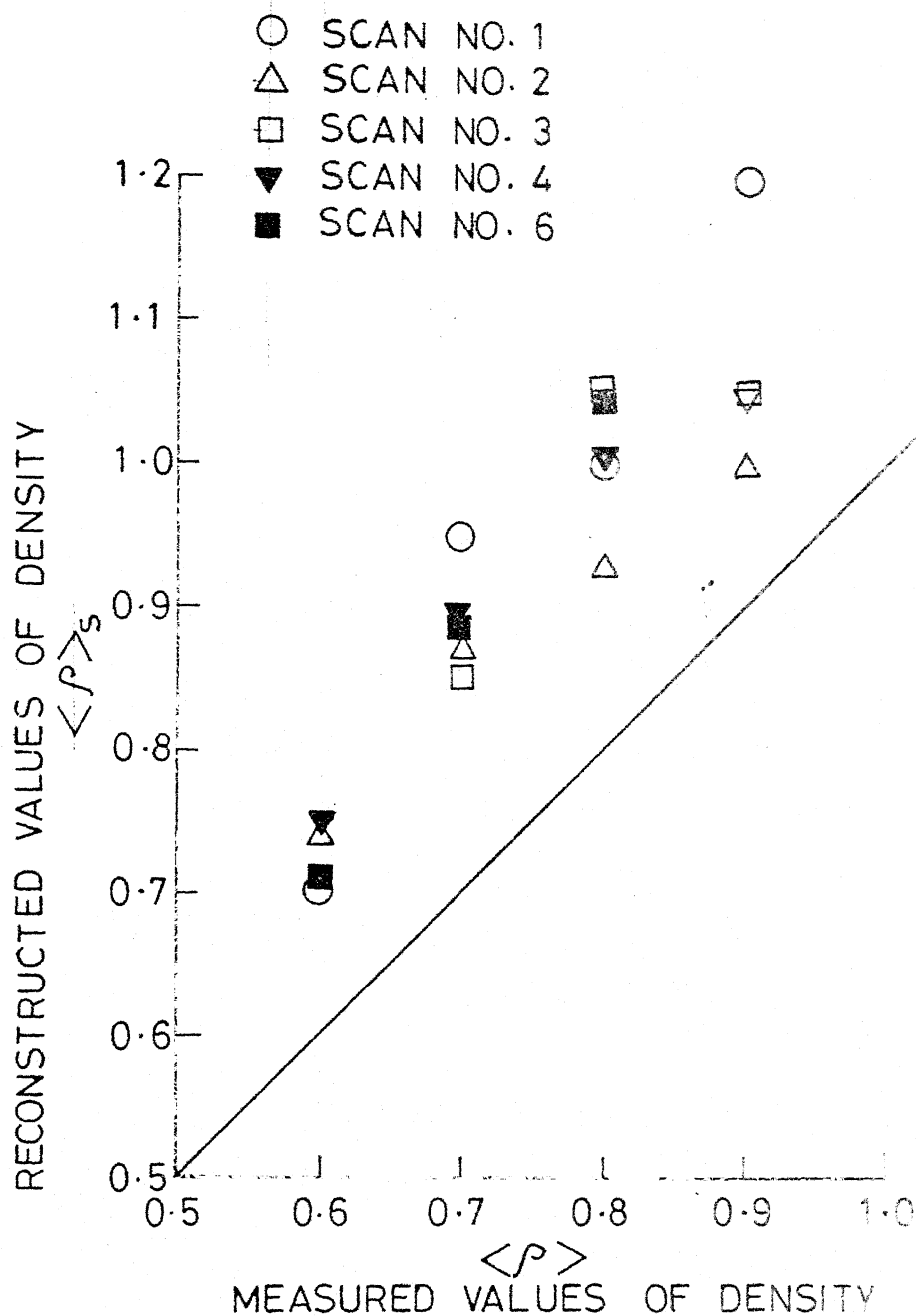


FIG. 4-7 COMPARISON BY SHEPP AND LOGAN FILTER OF ACTUAL DENSITY AND DENSITY FROM CAT SCAN

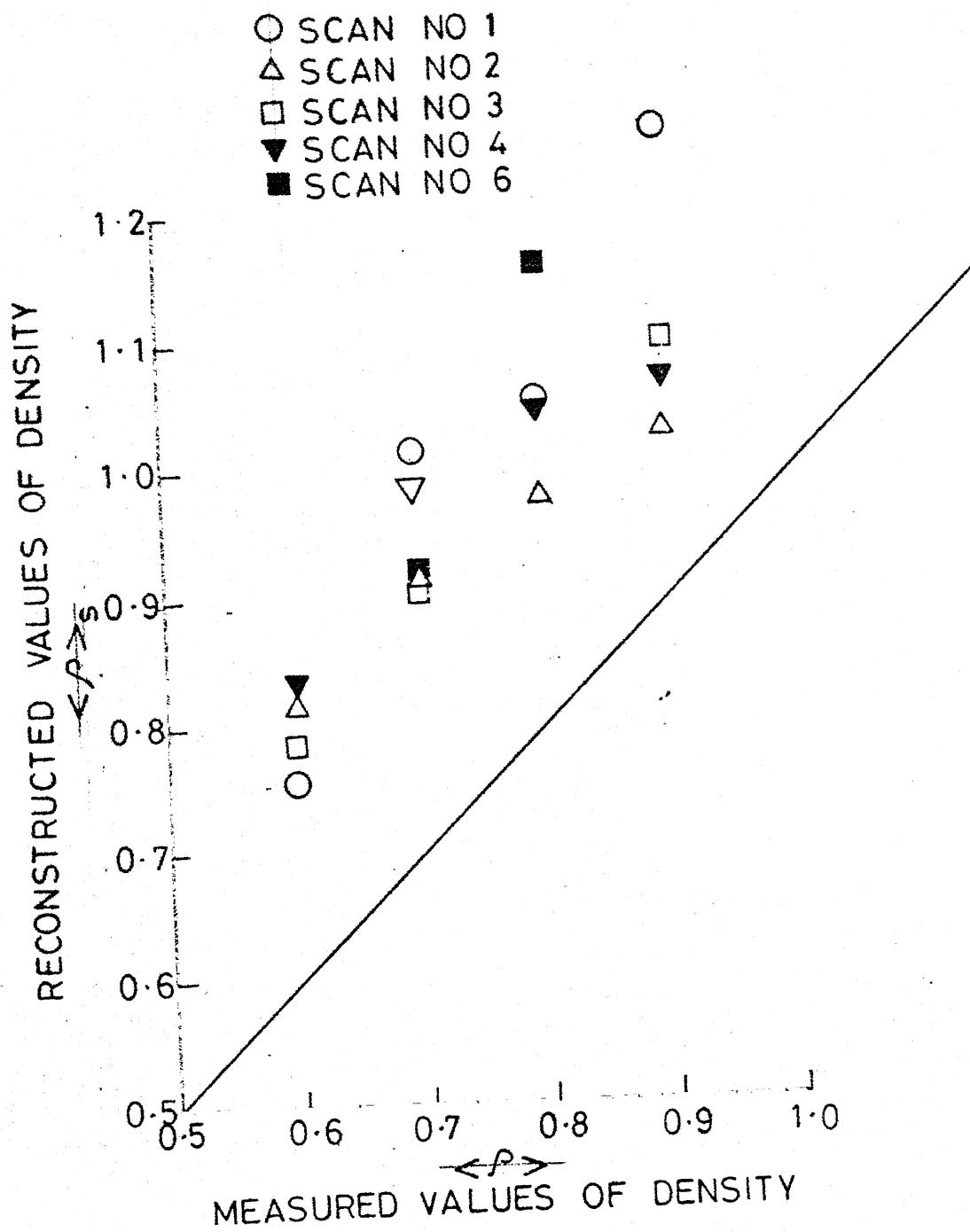


FIG. 4.8 COMPARISON BY PARABOLIC FILTER OF ACTUAL DENSITY AND DENSITY FROM CAT SCAN.

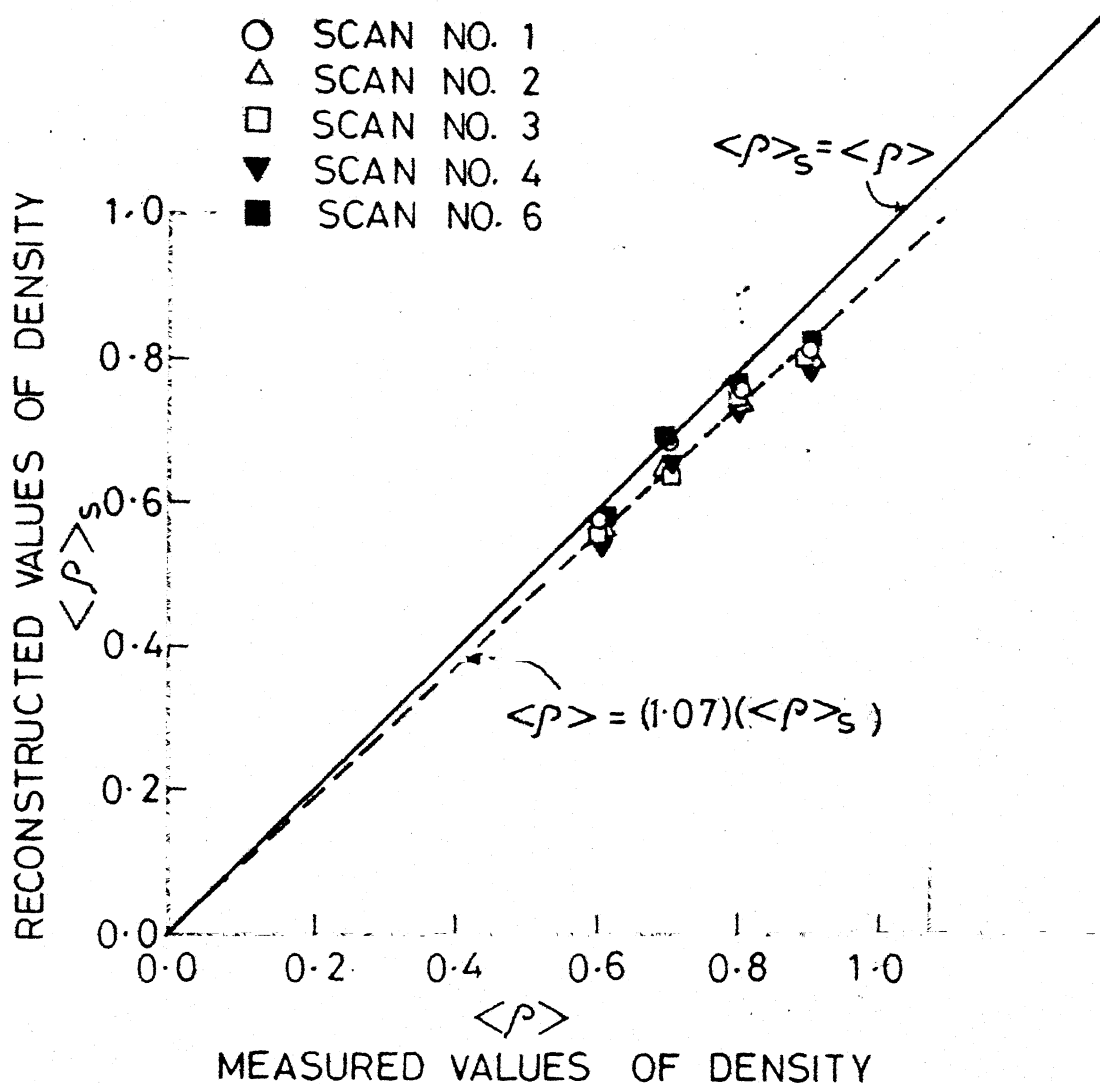


FIG. 4.9 COMPARISON BY GENERALISED FILTER NO.1 OF ACTUAL DENSITY AND DENSITY FROM CAT SCAN

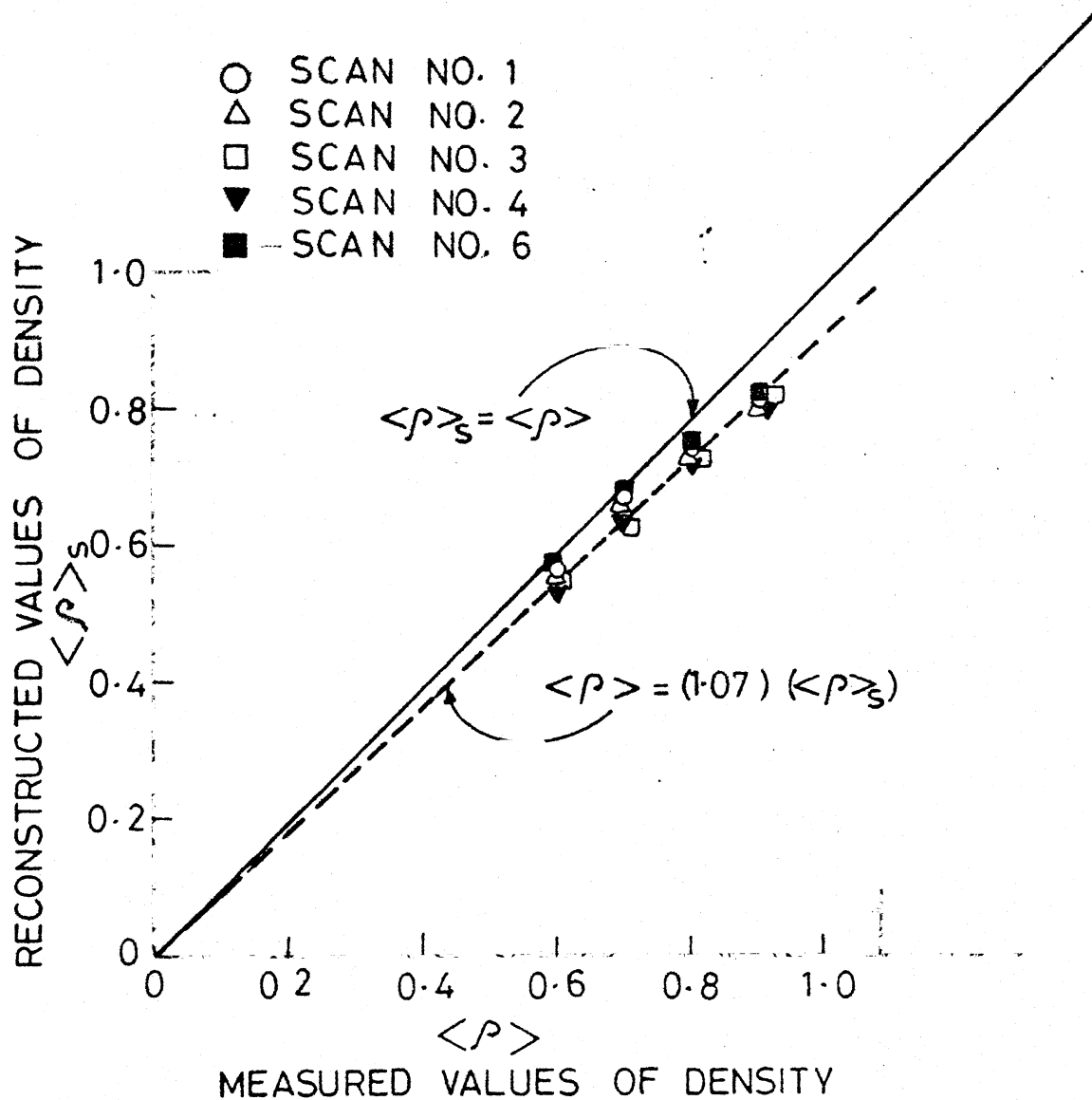
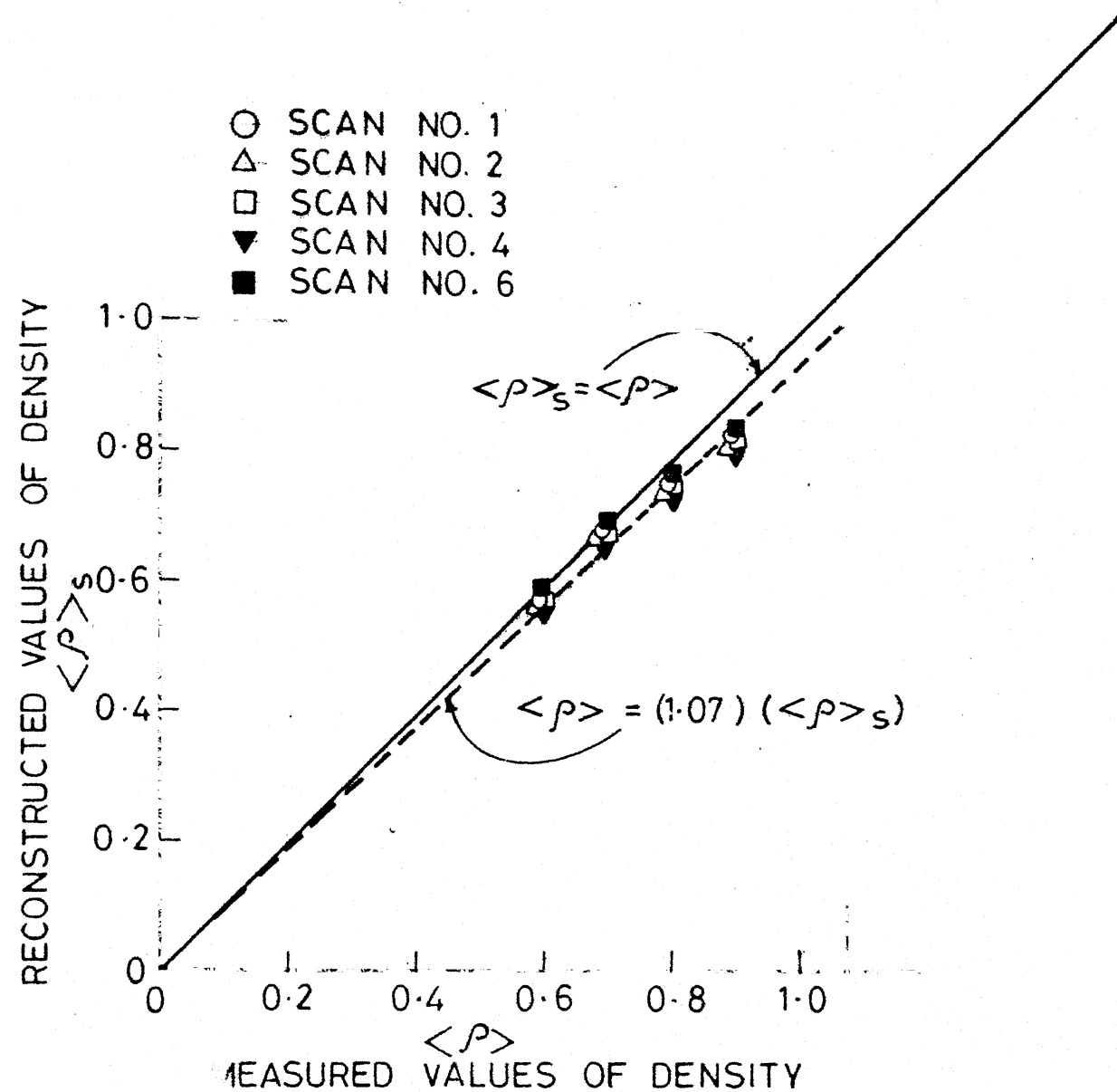


FIG. 4.10 COMPARISON BY GENERALISED FILTER NO.2 OF ACTUAL DENSITY AND DENSITY FROM CAT SCAN



G. 4.11 COMPARISON BY GENERALISED FILTER NO. 3 OF ACTUAL DENSITY AND DENSITY FROM CAT SCAN

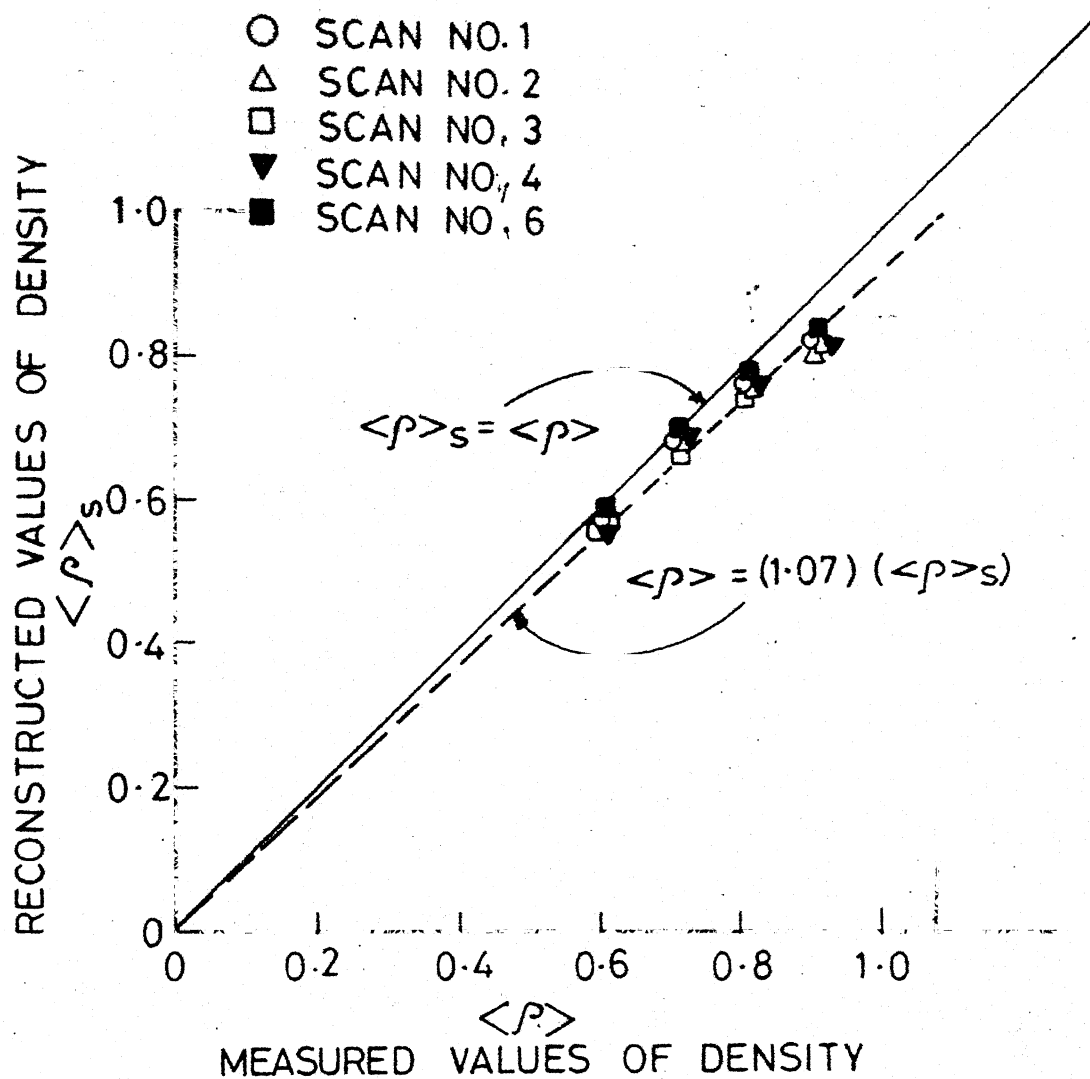


FIG. 4-12 COMPARISON BY GENERALISED FILTER NO. 4 OF ACTUAL DENSITY AND DENSITY FROM CAT SCAN

Tables (4.4 to 4.9) show the reconstructed density values for various scans and for various filters. Fig.(4.7) to (4.12) includes a comparison of the cross sectionally averaged values of density by the scan method to other method, denoted by $\langle \rho \rangle_s$ and $\langle \rho \rangle$ respectively. The darkline indicates a perfect match between the values of density measured by the two different methods. A noticeable feature in Fig. (4.7) and (4.8) is that the values of $\langle \rho \rangle_s$ are greater than the values of $\langle \rho \rangle$ for all the scans. However, for all the cases of the generalised filter (Fig. 4.9 to 4.12) we see a considerable improvement in the LITF values compared to those of the Shepp and Logan, and the parabolic filters.

Table (4.10) summarize the deviations of $\langle \rho \rangle_s$ values from the $\langle \rho \rangle$ values. Fig. (4.13) represent Table (4.10) in a graphical form. There is an underestimation of true values by the reconstructed density values as can be clearly made out in Figs.(4.9 to 4.12) . The difference $\delta \langle \rho \rangle$, between these two values follows a pattern with width of $\Delta \langle \rho \rangle$ value.

The average of the absolute deviations of the the values of $\langle P \rangle_s$ i.e, the average of $|\delta \langle P \rangle|$ for various scans is shown in Table (4.11) and graphically represented in Fig.(4.14). Also appearing are the relative errors,

$\frac{\Delta \langle P \rangle}{\langle P \rangle}$, which are plotted against ρ in Fig.(4.14). The quantity ρ is computed by the following equation.

$$\Delta \langle P \rangle = \sum_{i=1}^n |\delta_i \langle P \rangle| / n$$

where n is the total number of scans considered. Since the error band of $\langle P \rangle_s$ values remain fairly constant for all density runs, a decrease in the relative error with decreasing density is observed.

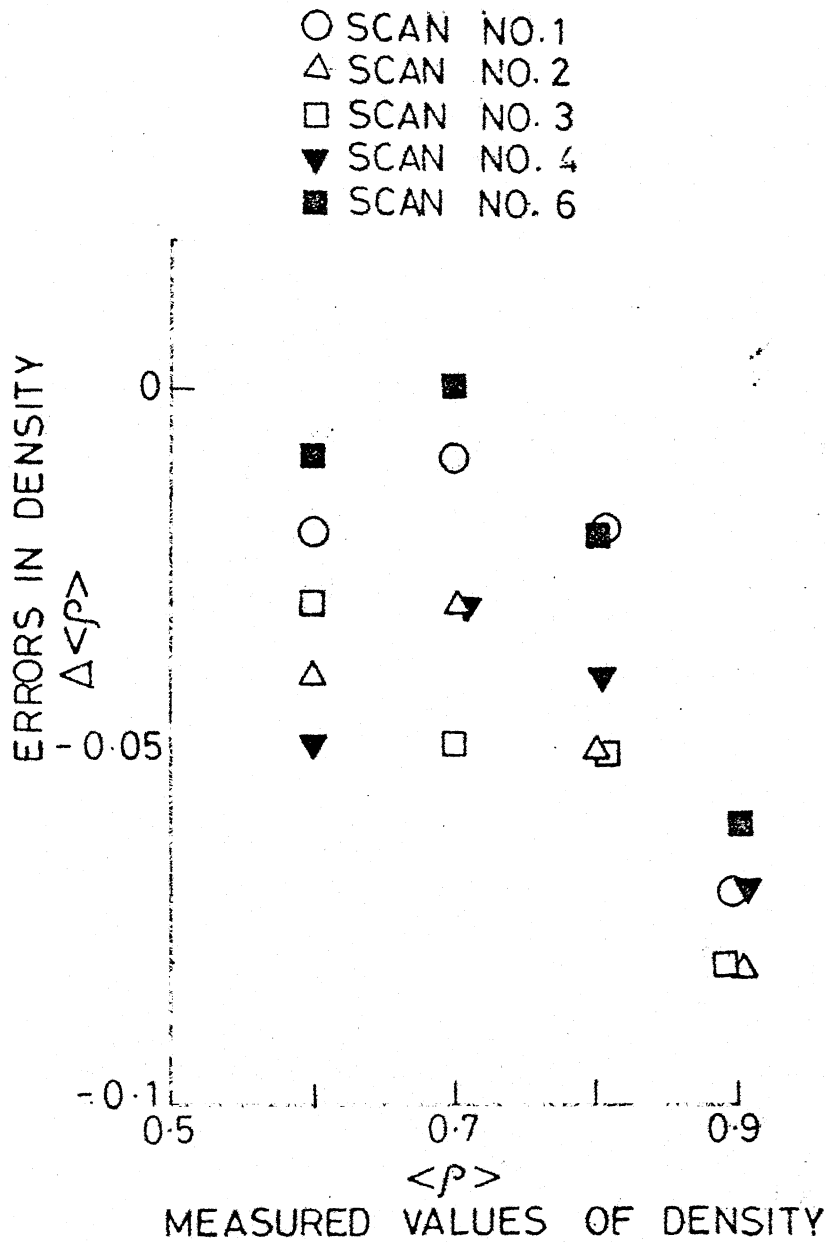


FIG. 4.13 ERROR IN DENSITY MEASUREMENTS FOR GENERALISED FILTER NO. 4

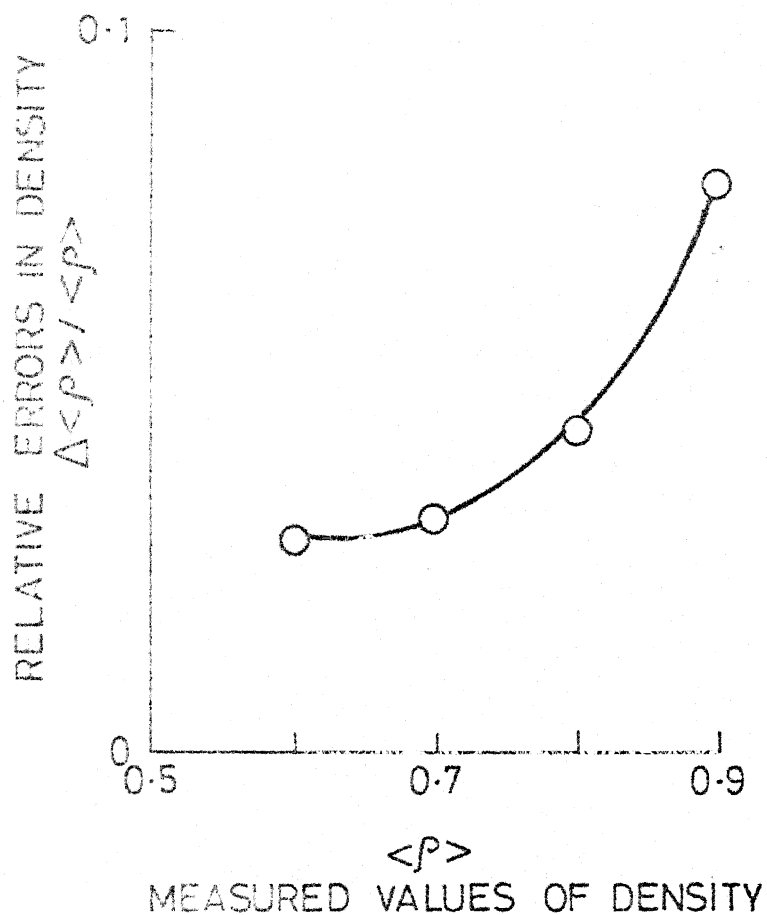


FIG.4.14 RELATIVE ERROR IN DENSITY MEASUREMENTS FOR GENERALISED FILTER NO. 4

CHAPTER-5

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions:

The following conclusions have been drawn based upon the results of the present study.

Meaningful results have been obtained using the techniques of computerised axial tomography (CAT) for density measurement in bubbly air-water flow. For cases investigated in the range $0.6 \leq \rho \leq 1.0$, the uncertainty band in the measured values of average density is found to be -0.03 , with respect to density determined experimentally by direct measurement. The density is underestimated for all scans. The dotted line in Fig(4.9 to 4.12) $\langle \rho \rangle = 1.07 \langle \rho \rangle_3$ clearly shows that the error can be overcome by a general approximation. This is true for all Generalised filters.

The results of density (scan) values using Shepp and Logan filter and Parabolic filter are enclosed to give a proper insight into the problem. The improvement the generalised filter offers can be highlighted by viewing Figures (4.1) to (4.12).

The above method is approximate and its limitations can be exposed by using less number of scans(N) and less number of rays per view (LITM). Proper care should be taken while choosing N and (LITM). The values of $N=20$ and $LITM = 40$ was used for the present study.

The method ('New Algorithm') is accurate and exact but takes longer time to evaluate. The method also removes the necessity of rotating the source provided the distribution is centrally or radially symmetric. However repeated scans at different scan angles will be required if the flow under investigation happen to be radially assymmetric.

5.2 Recommendation:

1. The Generalised filter used needs further investigation and selection of various values for ρ and ζ . The mathematics part of Generalised filter needs proper understanding.
2. The Noise error analysis as suggested by Kwok et al (7) is to be considered and this is an important factor.
3. Plexi glass was used in the set up for data calculation but its effect was neglected i.e., not taken into

account in the algorithm. Suitable modification in the algorithm is necessiated and 'New Algorithm' comes into prominence for this problem.

4. The 'New Algorithm' explained in section 6.0 has to be adapted for experimental data by suitable modifications.

CHAPTER - 6A NEW ALGORITHM

The Algorithm described and used in the present study is based on convolution i.e., filtered back projection. It is used primarily because of the high processing speed and reasonable engineering accuracy.

A " New Algorithm " is being proposed for the reconstruction studies in the two-phase flow area. To test the concept we generated data from a known density distribution, then used the general data to reconstruct the assumed density distribution. A brief summary of the procedure is given below:

Step 1 :- Data is represented as an array and given as a column of n elements where, each element is the chordal averaged value at different scan angles.

$$d = \begin{pmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{pmatrix}$$

We note that N or LITN explained before and defined as Number of scans is unnecessary in this case.

Step 2 :- We assume a function g , a polynomial of degree

$$\leq n \text{ in } r^2.$$

$$\text{where } g = x_1 + x_2 r^2 + \dots + x_m r^{2m-2}$$

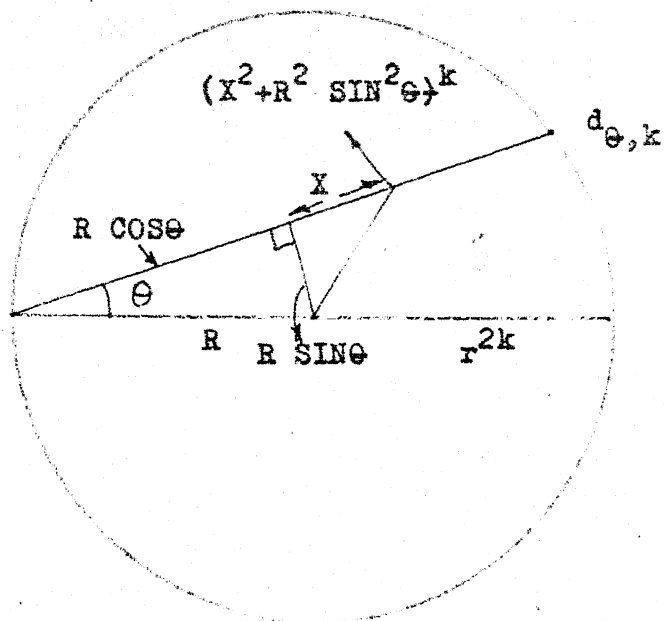


Fig. 6.1 Assumed distribution for New Algorithm.

where x is the coefficients of powers of r^2 .

x can be represented as

$$x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix}$$

Step 3 :- Now $JX = D$

(6.3)

where J is the weight of each part which when multiplied by x will give rise to d .

where

$$J_k^{(i)} = \frac{2R^2 \cos \theta_i}{2k-1} \left[R^{2k-3} + (k-1) \sin \theta_i \tan \theta_i J_{k-1}^{(i)} \right]$$

$$k = 2(1) m$$

$$i = 1(1) n$$

$$J_1^{(i)} = 2R \cos \theta_i, \text{ } i^{\text{th}} \text{ element of 1st column}$$

$$i = 1(1) n, \text{ This gives } n \times m \text{ dimension for } J.$$

for derivation of J see Appendix E.

On solving the system of linear equations $Jx = D$, we obtain the X -vector

Step 4:- The reconstructed value at the point r is given by

$$F(r) = \rho^T X = \sum_{i=0}^m r^{2i} x_i$$

where $\rho = \begin{pmatrix} 1 \\ r^2 \\ \vdots \\ r^{2m-2} \end{pmatrix}$

Step 5 :- The data used is generated especially to verify the 'New Algorithm'. The data element d is the chordal average and is calculated using the formule $d = 2 \int_0^{\cos \theta} f(r) dx$.

where $f(r) = f(\sqrt{x^2 + \sin^2 \theta})$ (from figure 6.1)

The accompanying data provide ample proof for the success of the algorithm.

A FORTRAN program has also been written and employed to give the best of results. NAG library routine has been used to perform Numerical Integration in calculating data and for solving system of linear equation to find X by Eqn (6.3).

Several functions of $f(r)$ were used to test the algorithm. Some typical cases are:

$$f(r) = \begin{cases} 1 \\ r \\ \exp(r) \\ \exp(-r) \end{cases}$$

Tables 6.1 to 6.4 show the reconstructed values, $F(r)$, and the exact values of the various function in the interval $0.0 \leq x \leq 1.0$ in steps of 0.1.

We observe that the new algorithm predicts the unknown function very accurately if it is radially symmetric. This algorithm has to be modified (a different J-Matrix has to be used) for use in radiation attenuation methods. Also, the algorithm can be extended for situation when the function is not radially symmetric. Such a case will involve multiple projections as currently being practised in CT algorithms.

TABLE 6.1

$$f(r) = 1.0$$

Sl No.	$f(r)$	$F(r)$	d
1	1.000	1.000	2.000
2	1.000	1.000	1.978
3	1.000	0.999	1.912
4	1.000	0.999	1.805
5	1.000	1.000	1.658
6	1.000	1.000	1.475
7	1.000	1.000	1.259
8	1.000	1.000	1.015
9	1.000	0.999	0.749
10	1.000	1.000	0.467
11	1.000	0.999	0.174

TABLE 6.2

$$f(r) = r$$

Sl No.	$f(r)$	$F(r)$	d
1	0.000	0.095	1.000
2	0.100	0.119	1.045
3	0.200	0.186	1.118
4	0.300	0.284	1.178
5	0.400	0.398	1.200
6	0.500	0.508	1.170
7	0.600	0.605	1.076
8	0.700	0.695	0.923
9	0.800	0.798	0.713
10	0.900	0.905	0.458
11	1.000	1.017	0.174

TABLE 6.3

$$f(r) = \exp(r)$$

Sl No.	f(r)	F(r)	d
1	1.000	1.095	3.436
2	1.105	1.124	3.473
3	1.221	1.207	3.518
4	1.345	1.334	3.522
5	1.491	1.489	3.450
6	1.649	1.657	3.273
7	1.822	1.827	2.967
8	2.014	2.008	2.521
9	2.226	2.224	1.941
10	2.460	2.465	1.246
11	2.718	2.735	0.473

TABLE 6.4

$$f(r) = \exp(-r)$$

Sl No.	f(r)	F(r)	d
1	1.000	0.967	1.264
2	0.905	0.915	1.205
3	0.819	0.816	1.091
4	0.741	0.739	0.944
5	0.670	0.673	0.811
6	0.607	0.605	0.671
7	0.549	0.549	0.536
8	0.497	0.497	0.409
9	0.449	0.449	0.289
10	0.406	0.406	0.175
11	0.368	0.368	0.064

```

01 c This program outputs data to the tomography system it accepts
02 c fan beam data for parallel to convert which converts the data
03 c to parallel beam.
04 c INTEGER M,LITN
05 c REAL ALPHA,D,LAMMAX,DELLAM,T,BETA,H,LAMBDA,THETA
06 c D is the distance from source to center of object
07 c D=4.0148734
08 c T is the diameter of the object
09 c T=6.5
10 c LITN is the number of projections in scan(fan beam)
11 c LITN=20
12 c M is the number of rays per view(fan beam)
13 c M=20
14 c Calculates maximum lambda
15 c LAMMAX=P/(2.0*SQRT(1.0-(T/(2.0*D))*2))
16 c Calculates ray spacing in terms of lambda
17 c DELLAM=2.0*LAMMAX/FLOAT(M)
18 c Calculates angular spacing between views
19 c ALPHA=3.1415927/FLOAT(ABS(LITN))
20 c Writing the output in file named input.dat
21 200 OPEN(UNIT=21,FILE='INPUT',ACCESS='SEQUENT')
22 WRITE(21,*)D,T
23 WRITE(21,*) LAMMAX,DELLAM,ALPHA
24 WRITE(21,*) M,LITN
25 CLOSE(UNIT=21)
26 TYPE 300
27 300 FORMAT(' NEXT PROGRAM IS CONVERT')
28 STOP
29 END

```

```

c This program converts fan beam data into parallel beam
c data. the data are then passed to the filter function
c Program (which is geometry dependent)
INTEGER M,LITN,LITM,N,K,J,S,I
REAL ALPHA,D,LAMMAX,LITA,DELLAM,LITH(41,41),T
REAL LITP(41,41),BETA,LAMBDA,LITL,THETA,GAMMA
OPEN (UNIT=21,FILE='INPUT.DAT',ACCESS='SEQIN')
5 FORMAT(E15.8)
c The data is read from file input.dat created by input.for
READ(21,*) D,T
READ(21,*) LAMMAX,DELLAM,ALPHA
READ(21,*) M,LITN
CLOSE (UNIT=21)
c N is the number of projections in scan (parallel)
N=20
c LITM is the number of rays per projection (parallel)
LITM=40
c FN gives the option to specify the required data file on
c The tty up to 5 letters maximum,e.g. water,wanut,etc.,
READ(5,9) FN
9 FORMAT(2A5)
DO 10 J=1,LITN+1
OPEN (UNIT=53,ACCESS='SEQIN',FILE=FN)
READ(53,5) (LITH(I,J),I=1,M+1)
CLOSE (UNIT=53)
10 CONTINUE
c LITH is the fan beam data as an array
DO 100 J=1,LITN+1
c The next three statements change LITH to density values
DO 100 I=1,LITM+1
LITH(I,J)=1.0-LITH(I,J)
100 CONTINUE
c LITA is the distance between parallel rays in each view
LITA=T/FLOAT(LITM)
c CONVERT TO PARALLEL
DO 80 K=1,LITM+1
LITL=LITA*FLOAT(K-1)-(T/2.)
GAMMA=ASIN(LITL/D)
LAMBDA=D*(SIN(GAMMA)/COS(GAMMA))
S=1+M/2+IFIX(FLOOR(LAMBDA/DELLAM))
IF (S.EQ.0) S=1
DO 70 J=1,N+1
THETA=(3.1415927/FLOAT(N))*FLOAT(J-1)
BETA=THETA+GAMMA
IF (BETA.LT.0.0) BETA=BETA+3.1415927
IF (BETA.GE.3.1415927) BETA=BETA-3.1415927
I=IFIX (FLOOR(BETA/ALPHA))+1
c LITP is the parallel beam data as an array
LITP(K,J)=LITH(S,I)
c Interpolation is being done in the next few statements
IF (S.LE.M) LITP(K,J)=LITP(K,J)+(LAMBDA/DELLAM-
1 FLOOR(LAMBDA/DELLAM))*(LITH(S+1,I)-LITH(S,I))
IF (I.LE.LITN) LITP(K,J)=LITP(K,J)+(BETA/ALPHA-FLOOR
1 (BETA/ALPHA))*(LITH(S,I+1)-LITH(S,I))
70 CONTINUE
80 CONTINUE

```

```

c      write geometry information in file named geom.dat
      OPEN (UNIT=23,FILE='GEOM',ACCESS='SEQOUT')
      WRITE (23,6) N,LITM
      WRITE (23,4) LITA
4      FORMAT(F15.9)
      CLOSE (UNIT=23)
c      write parallel data in file named convrt.dat
      OPEN (UNIT=24,FILE='CONVRT',ACCESS='SEQOUT')
      WRITE (24,6) N,LITM
6      FORMAT(I2,3X,I2)
      WRITE(24,7) ((LITP(J,I),J=1,LITM+1),I=1,N+1)
7      FORMAT(4(E15.8,3X))
      CLOSE (UNIT=24)
      TYPE 410
410    FORMAT ('-NEXT PROGRAM IS FILTER')
      STOP
      END
c      This function computes the greatest integer less than or
c      equal to the argument. it returns a real result.
      FUNCTION FLOOR(X)
      REAL X
      FLOOR=IFIX(X)
      IF (FLOOR.GT.X) FLOOR=FLOOR-1.0
      RETURN
      END

```



```

01 REAL LITP(41,41),LITW(41),SUM,LITC(41,41)
02 INTEGER LITM,N,J,K,L
03 10 FORMAT(I2,2X,I2)
04 c Read parallel data from convrt.dat created by convrt.for
05 OPEN (UNIT=24,FILE='CONVRT',ACCESS='SEQIN')
06 READ (24,*) N,LITM
07 3 FORMAT(I2,3X,I2)
08 READ (24,*) ((LITP(K,J),K=1,LITM+1),J=1,N+1)
09 5 FORMAT(4(E15.3,3X))
10 CLOSE (UNIT=24)
11 c Read fourier filter from filter.dat created by filter.for
12 OPEN (UNIT=51,FILE='FILTER',ACCESS='SEQIN')
13 c LITW is the filter function data
14 DO 30 K=1,LITM+1
15 READ (51,*) LITW(K)
16 40 FORMAT(E15.8)
17 20 FORMAT(4E15.8)
18 30 CONTINUE
19 CLOSE (UNIT=51)
20 c Compute convolution
21 DO 70 L=1,LITM+1
22 DO 60 J=1,N+1
23 SUM=0.0
24 DO 50 K=1,LITM+1
25 SUM=SUM+LITP(K,J)*LITW(ABS(L-K)+1)
26 50 CONTINUE
27 LITC(L,J)=SUM
28 60 CONTINUE
29 70 CONTINUE
30 c Write convoluted data in convlv.dat
31 OPEN (UNIT=52,FILE='CONVLV',ACCESS='SEQOUT')
32 c Writing the convoluted data
33 WRITE(52,*) N,LITM
34 DO 100 K=1,N+1
35 WRITE(52,*) (LITC(L,K),L=1,LITM+1)
36 100 CONTINUE
37 CLOSE (UNIT=52)
38 TYPE 200
39 200 FORMAT('-NEXT PROGRAM IS "SUPERIMPOSITION".')
40 STOP
41 END

```

```

01 REAL TRUET,LITF(41,41),LITC(41,41),SUM,T,THETA
02 INTEGER J,L,X,Y,LITM,N
03 c Read in convoluted data from convlv.dat created by convlv.for
04 OPEN (UNIT=54,FILE='CONVLV',ACCESS='SEQIN')
05 READ (54,*) N,LITM
06 2 FORMAT(I2,2X,I2)
07 c LITC is the convoluted data
08 DO 6 L=1,N+1
09 READ (54,*) (LITC(J,L),J=1,LITM+1)
10 4 FORMAT(4E15.8)
11 6 CONTINUE
12 CLOSE (UNIT=54)
13 c Calculate superimposition of data
14 DO 30 X=1,LITM+1
15 DO 20 Y=1,LITM+1
16 SUM=0.0
17 DO 10 J=1,N
18 THETA=(3.1415927)*FLOAT(J-1)/FLOAT(N)
19 TRUET=FLOAT(X-LITM/2-1)*COS (THETA)+FLOAT(Y-LITM/2-1)
20 1*SIN(THETA)
21 T=TRUET+FLOAT(LITM/2+1)
22 L=IFIX(FLOOR(T))
23 SUM=SUM+(FLOOR(T+1.0)-T)*LITC(L,J)+(T-FLOOR(T))*LITC(L+1,J)
24 10 CONTINUE
25 c LITF is the back-projected data as an array
26 LITF(X,Y)=SUM/FLOAT(2*N)
27 20 CONTINUE
28 30 CONTINUE
29 c Write output data in supimp.dat
30 LITM=LITM+1
31 OPEN (UNIT=55,FILE='SUPIMP',ACCESS='SEQOUT')
32 WRITE (55,2) LITM,LITM
33 WRITE(55,1) ((LITF(X,Y),X=1,LITM),Y=1,LITM)
34 1 FORMAT(4E15.8)
35 CLOSE (UNIT=55)
36 TYPE 1000
37 1000 FORMAT('-NEXT PROGRAM IS "PRINT"!')
38 STOP
39 END
40 FUNCTION FLOOR(A)
41 c Function is similar to the one explained before in convrt.for
42 REAL A
43 FLOOR=IFIX(A)
44 IF (FLOOR.GT.A) FLOOR=FLOOR-1.0
45 RETURN
46 END

```

```

01 LOGICAL EVEN
02 REAL LITF(41,41)
03 INTEGER M, LINE(41), XMAX, YMAX, SUPERI, PRINT, PRT, TITLE(20)
04 c Read data from supimp.dat created by supimp.for
05 OPEN(UNIT=55, FILE='SUPIMP', ACCESS='SEQIN')
06 READ(55,3) XMAX, YMAX
07 READ(55,1) ((LITF(I,J), I=1, XMAX), J=1, YMAX)
08 1 FORMAT(4E15.8, 2X)
09 3 FORMAT(12, 2X, I2)
10 CLOSE (UNIT=55)
11 c Print data
12 Y=9999.0
13 Z=-9999.0
14 DO 30 L=1, XMAX
15 DO 20 J=1, YMAX
16 IF(Y.GT.LITF(L,J)) Y=LITF(L,J)
17 IF(Z.LT.LITF(L,J)) Z=LITF(L,J)
18 20 CONTINUE
19 30 CONTINUE
20 TYPE 35
21 35 FORMAT('-NEGATIVE OF IMAGE(0=NO,1=YES)')
22 ACCEPT *,ANS
23 TYPE 36
24 36 FORMAT('PRINT IMAGE ON TERMINAL?(0=NO,1=YES)')
25 ACCEPT *,PRT
26 c Open print file
27 EVEN=.FALSE.
28 OPEN (UNIT=56, FILE='RESULT', ACCESS='SEQOUT')
29 DO 55 L=1, XMAX
30 DO 40 J=1, YMAX
31 LINE(J)=IFIX(0.5+99.0*(LITF(L,J)-Y)/(Z-Y))
32 IF(ANS.NE.1) LINE(J)=99-LINE(J)
33 40 CONTINUE
34 WRITE (56,45) (LINE(M), M=1, YMAX)
35 45 FORMAT(1H, 50I2)
36 IF (EVEN) GOTO 300
37 WRITE(56,200) (LINE(M), M=1, YMAX, 2)
38 WRITE(56,200) (LINE(M), M=1, YMAX, 2)
39 200 FORMAT(1H+, 25(I2, 2X))
40 GOTO 500
41 300 WRITE(56,400) (LINE(M), M=2, YMAX, 2)
42 WRITE(56,400) (LINE(M), M=2, YMAX, 2)
43 400 FORMAT(1H+, 25(2X, I2))
44 500 EVEN=.NOT.EVEN
45 IF (PRT.EQ.1) TYPE 45, (LINE(M), M=1, YMAX)
46 55 CONTINUE
47 WRITE(56,60) Y,Z
48 60 FORMAT(/, MINIMUM LITF=',E15.8,', MAXIMUM LITF=',E15.8)
49 IF (PRT.EQ.1) TYPE 60, Y,Z
50 70 FORMAT(/, -LITF=',E15.8)
51 CLOSE (UNIT=56)
52 STOP
53 END

```

```
01 c This program integrates the litt values printed out in
02 c Result.dat and gives a number for the 31x31 values.
03 c The number can be changed by using a higher value for the matri
04 c PARAMETER NN=41
05 c INTEGER ST(41),A(41,41),B(41,41)
06 c Input is taken from result.dat created by print2.for
07 c OPEN (UNIT=21,FILE='RESULT',ACCESS='SEQIN')
08 c READ(21,*) ((B(I,J),J=1,NN),I=1,NN)
09 c SUM=0.0
10 c NUM=0
11 c DO 10 I=5,35
12 c DO 20 J=5,35
13 c NUM=NUM+1
14 c SUM=SUM+B(I,J)
15 c CONTINUE
16 c AVRAGE=SUM/NUM
17 c The output is created in answer.dat
18 c OPEN(UNIT=23,FILE='ANSWER',ACCESS='SEQOUT')
19 c Open (unit=23,device='tty',access='seqout')
20 c WRITE(23,*) AVRAGE,SUM,NUM
21 c STOP
22 c END
```

```
01 C This program calculates the fourier filter function. this
02 C particular function is the parabolic filter function
03 REAL LITA,CONST,W(41)
04 INTEGER LITM,N,K,GEOM,FILTER
05 C Read geometry factors
06 OPEN (UNIT=23,FILE='GEOM',ACCESS='SEQIN')
07 READ (23,*)N,LITM
08 20 FORMAT(I2,3X,I2)
09 READ (23,*) LITA
10 10 FORMAT(E15.8)
11 CLOSE(UNIT=23)
12 C Calculates constant
13 W(1)=(3.1415927)/(3.*LITA*LITA)
14 CONST=-1.0/(3.1415927*LITA*LITA)
15 C Write filter values
16 OPEN (UNIT=51,FILE='FILTER',ACCESS='SEQOUT')
17 DO 30 L=1,LITM+1
18 W(L+1)=CONST/FLOAT((L)**2)
19 30 CONTINUE
20 WRITE (51,10) (W(L),L=1,41)
21 CLOSE (UNIT=51)
22 STOP
23 END
```

```

01  C These are filter values for Generalised Filter for Odd N's
02  C The values of Zeta and P are specified in the Program
03  C The Subroutine used is the NAG library routine D01AKF
04  C The routine is called twice for the two separate terms.
05  INTEGER IW(600)
06  DIMENSION W(2000),RESULT(42),VALUE(42),FINAL(42)
07  EXTERNAL F3,F4
08  COMMON N,P,ZETA,PIE
09  A=0.0
10  PIE=22./7.
11  B=2.*PIE
12  EPSABS=0.1E-01
13  EPSREL=0.1E-01
14  IFAIL=0
15  ZETA=4.75E-03
16  P=2.0
17  DO 10 N=3,41,2
18  C The routine below does the first part of integration
19  C this uses a function called f3 which is external
20  CALL D01AKF(F3,A,B,EPSABS,EPSREL,RESULT(N),ABSERR,
21  1 W,2000,IW,600,IFAIL)
22  10 CONTINUE
23  DO 11 N=3,41,2
24  C The routine below does the second part of integration
25  C this uses a function called f4 which is external
26  CALL D01AKF(F4,A,PIE,EPSABS,EPSREL,VALUE(N),ABSERR,
27  1 W,2000,IW,600,IFAIL)
28  2 FORMAT(E15.6)
29  FINAL(N)=RESULT(N)+VALUE(N)
30  11 FINAL(N)=FINAL(N)/(PIE*0.1625*0.1625*N*N)
31  WRITE(24,2) (FINAL(I),I=3,42,2)
32  WRITE(5,*) IFAIL,ABSERR,A,B
33  STOP
34  END
35  FUNCTION F3(X)
36  C This is used for the first part of integration
37  COMMON N,P,ZETA,PIE
38  SUM1=0.0
39  AS=0.1625
40  K=(N-3)/2
41  DO 100 J=1,K
42  Y=X+2.*PIE*FLOAT(J)
43  Y1=(Y/(AS*FLOAT(N)))*P
44  100 SUM1=SUM1+Y*EXP(-ZETA*Y1)
45  F3=SUM1*COS(X)
46  RETURN
47  END
48  FUNCTION F4(X)
49  C This is used for the second part of integration
50  COMMON N,P,ZETA,PIE
51  SUM2=0.0
52  AS=0.1625
53  YY=X+PIE*(FLOAT(N)-1.)
54  YY1=(YY/(AS*N))*P
55  SUM2=COS(X)*YY*EXP(-ZETA*YY1)
56  F4=SUM2
57  RETURN
58  END

```

```

01 C The filter values are the Generalised Filter for Even N's
02 C The values of Zeta and P are specified in the Program
03 C The Subroutine used is the NAG library routine D01AKF
04 INTEGER IW(600)
05 DIMENSION W(2000),RESULT(42)
06 EXTERNAL F2
07 COMMON N,P,ZETA,PIE
08 A=0.0
09 C A is the lower limit and B is the upper limit of the integral
10 PIE=22.77.
11 B=2*PIE
12 C EPSABS and EPSREL are the absolute and relative errors.
13 EPSABS=0.1E-01
14 EPSREL=0.1E-01
15 IFAIL=0
16 ZETA=4.75E-03
17 C Zeta is the damping coefficient
18 P=2.0
19 DO 10 N=2,40,2
20 CALL D01AKF(F2,A,B,EPSABS,EPSREL,RESULT(N),ABSERR,
21 1 N,2000,IW,600,IFAIL)
22 RESULT(N)=RESULT(N)/(PIE*0.1625*0.1625*N*N)
23 10 CONTINUE
24 WRITE(24,2) (RESULT(I),I=2,40,2)
25 C Result has the value of the integral
26 2 FORMAT(E15.6)
27 STOP
28 END
29 FUNCTION F2(X)
30 C These statements are the value of the integral
31 C employed in this Program,w.r.t. x
32 COMMON N,P,ZETA,PIE
33 SUM=0.0
34 AS=0.1625
35 K=N/2-1
36 ZN=N
37 DO 100 J=1,K
38 Y=X+2.*PIE*FLOAT(J)
39 Y1=(Y/(AS*ZN))*P
40 100 SUM=SUM+(Y*EXP(-ZETA*Y1))
41 F2=SUM*COS(X)
42 RETURN
43 END

```



```

01      C      These are the filter values of Generalised Filter for N=0
02      C      The values of Zeta and P are specified in the Program.
03      INTEGER IW(200)
04      REAL W(800)
05      EXTERNAL F0
06      A=0.0
07      B=22./((7.*0.1625)
08      EPSABS=0.1
09      EPSREL=0.1
10      IFAIL=0
11      CALL DO1AKF(F0,A,B,EPSABS,EPSREL,RESULT,ABSERR,
12      1 W,800,IW,200,IFAIL)
13      RESULT=RESULT*7./22.
14      WRITE(5,*) RESULT,ABSERR,A,B,IFAIL
15      STOP
16      END
17      C      FUNCTION F0(X)
18      C      These statements calculate the value of the integral
19      C      function used in the program,w.r.t. x
20      ZETA=4.75E-03
21      P=2.0
22      F0=X*EXP((- (ZETA)*X**P))
23      RETURN
24      END

```



```

01  C  These are the values of generalised filter for n=1
02  C  the values of zeta and p are specified in the program
03  C  the Subroutine used is the nag library routine dolakf
04  C  INTEGER IW(600)
05  REAL W(2000)
06  EXTERNAL F1
07  A=0.0
08  B=22./7.
09  EPSABS=0.1E-04
10  EPSREL=0.1E-02
11  IFAIL=0
12  CALL DOLAKF(F1,A,B,EPSABS,EPSREL,RESULT,ABSERR,
13  1 W,2000,IW,600,IFAIL)
14  RESULT=RESULT/(B*0.1625*0.1625)
15  WRITE(5,*) RESULT,ABSERR,A,B,IFAIL
16  STOP
17  END
18  C
19  C  These statements calculate the value of the integral
20  C  function used and says thatt is w.r.t. x
21  ZETA=4.75E-03
22  P=2.0
23  F1=X*COS(X)*EXP(-ZETA*(X/0.1625)**P)
24  RETURN
25  END

```

```
01  C These Program reads the values of Generalised filter's
02  C Odd and Even and puts them in a single file inserting
03  C alternate lines from the two inputs. The output is available
04  C in For51.dat and input through Odd.dat and Even.dat
05  OPEN(UNIT=21,ACCESS='SEQIN',FILE='EVEN')
06  OPEN(UNIT=22,ACCESS='SEQIN',FILE='ODD')
07  10 READ (21,*,END=20) A
08  WRITE (51,*) A
09  READ (22,*,END=20) A
10  WRITE (51,*) A
11  GOTO 10
12  20 STOP
13  END
```

Table B.7 Scan No. 1 - Normalized

DEGREES

TYPE SCAN	-30	-27.5	-25	-22.5	-20	-17.5	-15	-12.5	-10	-7.5	-5	-2.5	0	2.5	5	7.5	10	12.5	15	17.5	20	22.5	25	27.5
AIR	1.191	.740	.757	.881	.931	.955	.976	.979	.994	1.001	1.004	1.003	1.000	.997	.999	.992	.980	.969	.956	.928	.877	.788	.561	1.172
46% VOID	1.195	.766	.466	.435	.416	.396	.389	.388	.393	.401	.406	.405	.409	.408	.405	.406	.401	.382	.398	.405	.415	.448	.539	1.170
40% VOID	1.167	.761	.454	.408	.370	.358	.342	.335	.331	.339	.341	.340	.337	.340	.348	.339	.345	.337	.349	.360	.382	.424	.519	1.172
30% VOID	1.185	.739	.464	.356	.301	.267	.244	.235	.227	.218	.220	.216	.222	.223	.224	.234	.241	.244	.262	.276	.321	.386	.507	1.172
20% VOID	1.189	.747	.425	.331	.267	.220	.190	.176	.156	.151	.147	.146	.145	.148	.156	.158	.172	.189	.215	.256	.292	.364	.498	1.166
10% VOID	1.197	.725	.428	.318	.242	.191	.148	.122	.097	.086	.081	.076	.086	.086	.091	.102	.119	.140	.168	.217	.267	.353	.499	1.153
WATER	1.200	.740	.422	.294	.205	.143	.097	.066	.040	.023	.010	.004	.000	.003	.014	.028	.048	.077	.118	.170	.231	.346	.517	1.181
WALNUT ($\rho = .732$)	1.184	.751	.561	.462	.397	.365	.333	.294	.261	.234	.224	.213	.216	.206	.209	.219	.236	.261	.303	.350	.402	.492	.540	1.179
PINE ($\rho = .41$)	1.185	.739	.578	.628	.593	.567	.540	.525	.503	.490	.482	.481	.478	.478	.487	.493	.506	.520	.546	.565	.583	.625	.543	1.172

Table B.9 Scan No. 3 - Normalized

DEGREES

[illegible]

Table B.10 Scan No. 4 - Normalized

[illegible]

Table B.11

DEGREES

MINIMUM LITF= 0.68083083E+00 MAXIMUM LITF= 0.26152078E+01

LITF=1.075

[The page contains dense vertical columns of numbers, likely representing data or code.]

MINIMUM LITF= 0.34922565E+00 MAXIMUM LITF= 0.18410774E+01

These are the values for Calibration of Pine ($p=0.41$) density for Shepp and Logan Filter.

$$\text{LITF}=0.351$$

MINIMUM LITF=-0.14364844E+00 MAXIMUM LITF= 0.18215783E+01

9960-0=JLIT

MINIMUM LITF= 0.68131146E+00 MAXIMUM LITF= 0.22452025E+01

$$LITF=0.928$$

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MINIMUM LITF= 0.46885256E+00 MAXIMUM LITF= 0.17809863E+01

These are the values for Calibration of Walnut(p=0.73) density for Generalised Filter No.1

LITF=0.740

[illegible]

MINIMUM LITF= 0.34911513E+00 MAXIMUM LITF= 0.15550466E+01

These are the values for Calibration of Pine($p=0.41$) density for Generalised Filter No. 4

These are the values for Calibration of Air ($p=0.0$) density for Generalised Filter No. 4

LITF=0.00

MINIMUM LITF=-0.15849042E+00 MAXIMUM LITF= 0.14964145E+01

[illegible]

MINIMUM LTF= 0.51906198E+00 MAXIMUM LTF= 0.25836699E+01

These are the values for Scan 6 of 90%(p=0.9) density for Shepp and Logan filter.

LITF=1.230

MINIMUM LIFE= 0.51920851E+00 MAXIMUM LIFE= 0.24996393E+01

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[illegible]

MINIMUM LTF= 0.49546118E+00 MAXIMUM LTF= 0.24181408E+01

These are the values for Scan 6 of 70% (p=0.7) density for Shepp and Logan Filter.

98.0317

MINIMUM LITF= 0.39276170E+00 MAXIMUM LITF= 0.22719101E+01

$$\text{LITF} = 0.73$$

MINIMUM LITF= 0.39471290E+00 MAXIMUM LITF= 0.19542150E+01

These are the values for Scan 6 of 60%(p=0.6) density for Generalised filter.

$$LTF = 0.59$$


```

C      This Program Generates the Data that is to be employed
C      in the 'New Algorithm' for testing it. This uses a NAG
C      library routine D01AKF which does Integration over any
      defined limits. The function is to be specified by the us
EXTERNAL F
DIMENSION W(800),IW(200),RESULT(11)
COMMON TH(11)
OPEN(UNIT=24,ACCESS='SEQIN',FILE='THETA')
READ(24,*)( TH(I),I=1,11)
A=0.0
DO 10 I=1,11
  B=COSD(TH(I))
  EPSREL=0.00001
  EPSABS=0.00001
  CALL D01AKF(F,A,B,EPSABS,EPSREL,RESULT(I),
10      1  ABSERR,W,800,LW,200,IFAIL)
  CONTINUE
DO 29 I=1,11
29      RESULT(I)=2.0*RESULT(I)
  OPEN(UNIT=21,ACCESS='SEQOUT',FILE='DATA')
  WRITE(21,9)(RESULT(I),I=1,11)
9      FORMAT(11E11.5)
  STOP
END
FUNCTION F(X)
COMMON TH(11)
Y=SIND(TH(I))
F2=SQRT(X*X+Y*Y)
F=EXP(-F2)
RETURN
END

```

C
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```

This Program Generates the Data for the 'New Algorithm'.
The NAG library routine used is the D01AHF routine.
The Function is to be specified by the user.
PROGRAM FOR INTEGRAL EQN
REAL A,B,EPSR,RELERR,TH,DT
INTEGER IFAIL,NLIMIT,N
REAL D01AHF
EXTERNAL FUN
COMMON TH
A = 0.0
NLIMIT = 0
EPSR = 1.0E-5
IFAIL = 0
DT = 8.5
DO 10 I = 1,11
TH = FLOAT(I-1)*DT
B = COSD(TH)
ANS = D01AHF (A,B,EPSR,N,RELERR,FUN,NLIMIT,IFAIL )
ANS=2.*ANS
WRITE (21,9) ANS
9    FORMAT(11E11.5)
10   CONTINUE
STOP
50   FORMAT (5X,'THETA=',F4.1,5X,'INTEGRAL=',F8.6,10X,
1    'ESTIMATED ERROR=',E15.5,3X,'IFAIL=',F5.3)
END
REAL FUNCTION FUN(X)
REAL X,K
COMMON TH
K = SIND(TH)
C    FUN = EXP(-(SQRT((X*X) + (K*K))))
fun=1.0
RETURN
END

```

```

C      This Program Generates Data for specific  $f(r)=r$  in
C      order to test the 'New Algorithm'. This doesnot utilise
C      the NAG library routine.
      dimension th(11),r(11)
      open(unit=21,access='segin',file='ttheta')
      read(21,*) (th(i),i=1,11)
      do 10 i=1,11
      y=(1+cosd(th(i)))/sind(th(i))
10     r(i)=cosd(th(i))+(((sind(th(i))*sind(th(i)))*alog(y))
      open(unit=23,access='segout',file='newdat')
      write(23,9) (r(i),i=1,11)
9      format(11e11.5)
      stop
      end

```

C This Program solves for X using the relation $JX=0$
 C Before this J is calculated using the formula for J
 C This Program also includes printing of f(r) i.e.,
 C Function Values and errors in Reconstruction E.

```
DIMENSION WKS(22),A(11,11),D(11),TH(11),R(11)
DIMENSION RV(11),C(11,11),Z(11),F(11),W(44),X(11)
LOGICAL SVD
N=11
```

```
OPEN(UNIT=21,ACCESS='SEQIN',FILE='THETA')
OPEN(UNIT=22,ACCESS='SEQIN',FILE='POINTS')
OPEN(UNIT=23,ACCESS='SEQIN',FILE='DATA')
```

```
READ(21,*) (TH(I),I=1,11)
READ(22,*) (Z(K),K=1,11)
READ(23,*) (D(J),J=1,11)
```

```
DO 10 I=1,11
Q=COSD(TH(I))
C(1,I)=2.*Q
DO 20 J=2,N
C(J,I)=((2.*Q)/(2.*(FLOAT(J))-1.))*(1.+
1 ((J-1)*(1.-Q*Q)/Q)*C(J-1,I))
20 CONTINUE
10 CONTINUE
```

```
TOL=5.*10**(-4)
DO 19 K=1,11
R(K)=1.0
DO 29 I=1,11
DO 29 J=1,N
29 A(I,J)=C(J,I)
OPEN(UNIT=23,ACCESS='SEQOUT',FILE='RESULT')
WRITE(23,9) A
IF=0
CALL F04JGF(11,11,A,11,D,TOL,SVD,SIGMA,IRANK,W,44,IF)
```

```
X=D
WRITE(5,*) SVD,D,IRANK
8 FORMAT(1X,4E10.4)
DO 50 I=1,11
DO 60 J=1,11
Y=Z(I)*Z(I)
RV(I)=(RV(I)*Y)+D(12-J)
```

```
60 CONTINUE
50 CONTINUE
DO 59 I=1,11
E(I)=RV(I)-R(I)
```

```
WRITE(23,9) X
WRITE(23,9) R,E
C This is the desired Reconstruction output
C RV represents the Reconstruction
OPEN(UNIT=55,FILE='OUTPUT',ACCESS='SEQOUT')
WRITE(55,7)(RV(J),J=1,11)
9 FORMAT(11E11.5)
7 FORMAT(1E11.5)
CLOSE(UNIT=55)
STOP
END
```

```

01      C      This Program checks the accuracy of solving JX=D
02      C      using NAG library routine by recalculating D.
03      DIMENSION A(11),B(11,11),C(11),Z(11)
04      OPEN(UNIT=23,ACCESS='SEQIN',FILE='RESULT')
05      READ(23,9) ((B(I,J),I=1,11),J=1,11)
06      READ(23,9) (C(I),I=1,11)
07      DO 10 I=1,11
08      DO 10 J=1,11
09      A(I)=A(I)+B(I,J)*C(J)
10      CONTINUE
11      OPEN(UNIT=24,ACCESS='SEQOUT',FILE='DSTAR')
12      WRITE(24,9) A
13      9      FORMAT(11E11.5)
14      STOP
15      END

```

```

01      C      This Program Regenerates the Data and uses it to
02      C      check the accuracy of Reconstruction.
03      EXTERNAL F
04      DIMENSION W(800),IW(200),RESULT(11)
05      COMMON TH(11),S(11)
06      OPEN(UNIT=22,ACCESS='SEQIN',FILE='THETA')
07      OPEN(UNIT=23,ACCESS='SEQIN',FILE='OUTPUT')
08      READ(22,*)( TH(I),I=1,11)
09      READ(23,*)( S(I),I=1,11)
10      A=0.0
11      DO 10 I=1,11
12      B=COSD(TH(I))
13      EPSREL=0.01
14      EPSABS=0.01
15      CALL D01AJF(F,A,B,EPSABS,EPSREL,RESULT(I),
16      1 ABSERR,W,800,LW,200,IFAIL)
17      CONTINUE
18      DO 29 I=1,11
19      29 RESULT(I)=2.0*RESULT(I)
20      OPEN(UNIT=20,ACCESS='SEQOUT',FILE='BACDAT')
21      WRITE(20,9)(RESULT(I),I=1,11)
22      9 FORMAT(11E11.5)
23      STOP
24      END
25      FUNCTION F(X)
26      COMMON TH(11),S(11)
27      K=10*X
28      J=K+1
29      F=S(J)+(10.*X-K)*(S(J+1)-S(J))
30      RETURN
31      END

```

APPENDIX-E

Referring to the figure we can write,

$$\begin{aligned}
 d_{\theta,k} &= 2 \int_0^{R \cos \theta} (x^2 + R^2 \sin^2 \theta)^k dx \\
 &= 2 (x^2 + R^2 \sin^2 \theta)^k x \Big|_0^{R \cos \theta} - k \int_0^{R \cos \theta} (x^2 + R^2 \sin^2 \theta)^{k-1} (2x^2) dx \\
 &= 2 \left[R^{2k+1} \cos \theta - 2k \int_0^{R \cos \theta} (x^2 + R^2 \sin^2 \theta)^{k-1} dx \right. \\
 &\quad \left. + 2k R^2 \sin^2 \theta \int_0^{R \cos \theta} (x^2 + R^2 \sin^2 \theta)^{k-1} dx \right] \\
 d_{\theta,k} &= 2R^{2k+1} \cos \theta - 2k d_{\theta,k} + 2k R^2 \sin^2 \theta d_{\theta,k-1} \\
 &= \frac{1}{2k+1} \left[2R^{2k+1} \cos \theta + 2k R^2 \sin^2 \theta d_{\theta,k-1} \right] \\
 &= \frac{2R^2 \cos \theta}{2k+1} R^{2k-1} + k \sin^2 \theta d_{\theta,k-1}
 \end{aligned}$$

For ease in FORTRAN programming,

$$J_k^{(i)} = \frac{2R^2 \cos \theta_i}{2k-1} \left[R^{2k-3} + (k-1) \sin^2 \theta_i \tan^2 \theta_i J_{k-1}^{(i)} \right]$$

$$J_1^{(i)} = 2R \cos \theta_i$$

$$k = 2(1) - n$$

$$i = 1(1) - n$$

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